Functions and Modeling ARC on Function Transformations

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Proposal: Composition of Functions ARC

Designated Course
Functions and Modeling

Targeted Topics
Composition of Functions

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Description of the ARC

In a Functions and Modeling course, the primary focus is on understanding functions, especially as they relate to modeling real-world scenarios. While it's relatively straightforward to grasp function composition by studying domains, ranges, and symbolic manipulation in abstract mathematical contexts, applying these concepts to models involving units and measurable quantities demands a more thorough analysis. This activity is designed to guide students through a series of scaffolded tasks that deepen their understanding of function composition, helping them critically evaluate how different contexts influence function composition and recognize meaningful applications.

Instructor Notes

TITLE OF LESSON: Function Composition

ESTIMATED TIME FOR LESSON (IN MINUTES): 1-2 hours

SUGGESTED FORMAT (check all that are appropriate):

- ☑ Individual in-class
- ☑ Collaborative in-class
- ☐ Individual homework
- ☐ Collaborative homework

OVERVIEW:

Effectively constructing and using function composition involves nuances that students can easily miss. This series of activities is designed to develop a conceptual understanding of the contexts in which compositions of functions do and do not have valid or desired results, the role units play in practical contexts involving function composition, and how domain and range relate to building the composition of functions. These activities can be used both to introduce function composition in a Functions and Modeling class, as well as to promote deeper student engagement and foster the students' abilities to understand function composition in both abstract and practical contexts. Moreover, the questions and contexts developed in these activities are designed to help students engage productively in the broader academic community.

Learning Outcomes: Students will be able to:

1. Identify when function compositions can and cannot be constructed using contextualized examples.
2. Identify when function compositions can and cannot be constructed using characteristics of the individual functions' domains and ranges.
3. Understanding the role of units in function composition.
4. Build appropriate compositions to solve problems with units.
5. Express function compositions with accurate notation.

PREREQUISITE IDEAS AND SKILLS:
Prerequisite and Corequisite Topics Developed

The following are prerequisite (or corequisite) conceptual understandings that are developed in prior (or related) exercises. Each concept is marked with P or C to indicate a prerequisite or corequisite, respectively.

- P) Evaluate functions algebraically - as plugging in an input x into an expression to yield output f(x).
- P) Identify the domain of functions.

- P) Evaluate the composition of functions algebraically - first evaluating function output g(x) and plugging this into function f to find f(g(x)).
- P) Evaluate compositions of functions using tables.
- C) Function decomposition
- C) Finding the domain of f(g(x))
- C) Mixing function representations (e.g f(x) is a graph and g(x) is a table)

MATERIALS NEEDED TO CARRY OUT THE LESSON:

- Calculator (physical or online)

These activities can be used independently. They are designed to progressively build the logical skill of identifying the ways the composition of functions can and cannot yield valid or desired results.

Activity 1 (Shape Shifting)

Description:

Activity 1 offers a visually accessible entry point for students to build an understanding of functions mapping various entities of allowed classifications. This builds the foundational logical skill of constructing possible compositions and evaluating when they satisfy given “rules” for valid constructions.

Activity 2 (International Business Deal)

Description:
Activity 2 develops a scenario in which currency conversions are needed to compare costs for a business deal. In this activity, students are motivated to make a series of independent conversions (procedurally and algebraically) to yield a composite function.

Notes:

Students may struggle to parse conversions and keep track of all them simultaneously. To help develop organizational cognitive structures for using the conversions effectively, instructors may choose to:

1. Omit the conversion formulas (text colored in dark gray) from the activity and construct them as a class based on the equivalency. Choice of variable and function notation can be discussed/emphasized.
2. Use a table to organize work and results. For example, draw 4 columns, one for each currency, and systematically record known values, formulas, and calculations in the appropriate columns.
3. Use spreadsheets (as a class, in groups, or individually) to document and tabulate the results of each part.

If students choose to use unit analysis to solve this activity, this can provide a productive opportunity to ask students to explain alternative approaches, and then use their explanations to highlight how a sequence of unit conversions can be conceptualized as a sequence of function compositions! The primary purpose of the activity is to develop an understanding of a context in which function composition is useful.

For time, question 5 (or questions 3-5) of Activity 2 can be omitted, though question 5 provides students the opportunity to approach a problem in different ways and compare results.

Activity 3 (Numerical Function Composition)

Description:
Activity 3 uses functions expressed as tables to study potential composition, f(g(x)).

Notes:

- Part I develops an example in which the range of g perfectly matches the domain of f, concluding that the function composition exists, and constructing a new table for the composite function.
- Part II highlights an example in which the range of g does not intersect the domain of f, concluding the composition does not exist.
- Part III provides an example in which some but not all of the range of g intersects the domain of f, concluding the composite function exists.

Activity 4 (Numerical Function Composition with Units)

Description:
Activity 4 provides an activity similar to Activity 3, but with the complexity being the units involved.

Notes:

- Part I provides a table in which the values and units pose no problem for f(g(x)) to exist with the composition function table being constructed
- Part II gives a table where the units pose a problem for f(g(x)) to exist.
- Part III gives a table in which the numerical values present no problem in the existence of either f(g(x)) or g(f(x)), but the units do for one of those. The numerical values in this case are of no importance as this part is presented as a means of scaffolding and creating continuity into the next and final activity.

Activity 5 (Function Composition with Words)

Description:

Activity 5 provides an activity like activity 3, but the representation for the function is provided in words as opposed to tables

Notes:

- Part I provides two functions in a context with units and asks students to simultaneously determine an appropriate composition order, if the value provided is an input or output, and if the value desired is an input or output.
- Part II provides two functions in a context with units, prompting students to determine which composition order makes sense and to identify the input and output units.
- Part III provides a scenario with two functions in a context with units in which only one ordering of function composition makes sense (both in terms of units and the order in which decisions are made). Students will have to decide which composition is logical, determine the input and output units of the composition, and then use an instance of the contextualized example to decide which pairs of function values make sense.