Lesson 5

TITLE OF LESSON: Applications of Transformations

ESTIMATED TIME FOR LESSON (IN MINUTES): 50 minutes

SUGGESTED FORMAT (check all that are appropriate):

Individual in-class
Collaborative in-class
Individual homework
Collaborative homework

OVERVIEW:

- Students use what they have learned in previous lessons to apply transformations to real-world scenarios.

PREREQUISITE IDEAS AND SKILLS:

- Students should be able to factor perfect squares in quadratic equations.
- The first problem could be used to motivate the need for completing the square, or completing the square could be prerequisite knowledge.
- Students should understand what a transformation is and be able to apply multiple transformations (shifting stretching, reflection) to a parent function.

MATERIALS NEEDED TO CARRY OUT LESSON:

- Students access to Internet for students' collaborative exploration
- "Applications of Transformations Worksheet" and Answer Key.

CONCEPTS TO BE LEARNED/APPLIED:

Guided exploration will help students to:

- Apply horizontal shifts, vertical shifts, horizontal or vertical stretches or compressions, and horizontal or vertical reflections to solve real-world applications.

<u>Teacher-Led, Student Work</u>

- Have students form pairs (or groups of three).
- Provide the following scenario. The instructor will work through it, but the students must discuss in their pairs and then as a class what was done.
- *<u>Scenario:</u>*
  When objects are thrown up, they slow down as they rise, until they reach their peak, then they start falling. When close to the surface of the Earth, the acceleration of gravity is about 32 feet per second. So, for every second an object falls without resistance, its speed increases by about 32 feet per second. This behavior is often modeled by

quadratic equations. For example, the equation $h = -16t^2 + 32t + 9$ models the height of a ball that is thrown upward out of a window, where t is in seconds and h is in feet. We will complete the square to get the equation into the form that shows the transformations that have occurred from a parabola in standard form.

- Instructor writes this step on the board then has the pairs discuss what was done at each step while circulating.

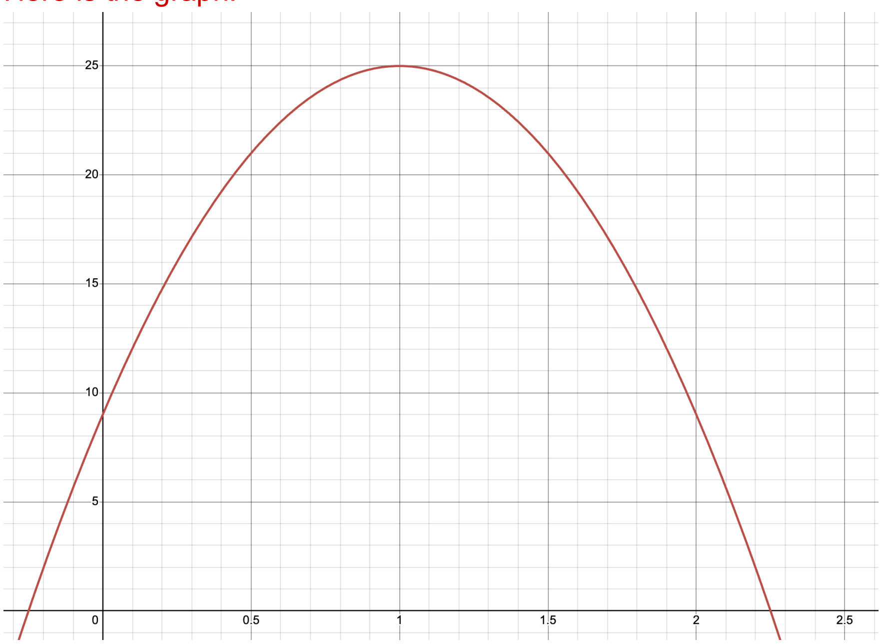
$$
\begin{aligned}
h &= -16t^2 + 32t + 9 \\
&= -16(t^2 - 2t) + 9 \\
&= -16(t^2 - 2t + 1 - 1) + 9 \\
&= -16(t^2 - 2t + 1) + 16 + 9 \\
&= -16(t - 1)^2 + 25
\end{aligned}
$$

- Instructor has 4 students (all from different pairs) each explain what has happened at a single step respectively.
- The students in their pairs then graph the parabola to answer these questions while the instructor circulates about the classroom.

  Questions:

  1. How high was the window from where the ball was thrown?
     Answer: 9 feet (this is the constant in the initial equation)
  2. What was the ball's peak height? When did this occur?
     Answers: 25 feet, 1 second after the ball was thrown (this is the vertex)
  3. How long did it take the ball to hit the ground?
     Answer: 2.25 seconds (this is the value of t when h = 0)

  Here is the graph:



In-Class Worksheet

The instructor should group the existing pairs to form slightly larger groups of about 4 students in each group to work on the following worksheet. Distribute the Applications of Transformations Worksheet. Students will be working with Desmos to complete the worksheet. This can be done in a guess-and-check mode based on their knowledge of transformations, or the students could also use sliders like they did in Lesson 4, even working with the same file but changing the parent function.

The first task involves a quadratic equation that models the firing of a missile from a helicopter. So, it is rather similar to the previous activity the students worked; however, the scale of numbers is quite different.

The second task involves a completely new function (a logistic function) and a scenario involving modeling the number of fish in a pond using Desmos and what students know about transformations. The instructor might want to use only Task 2a, which might be slightly easier (and is easier to grade) or only Task 2b, which is a bit more challenging. However, if there is time, the instructor could use both.

Reflection (after completion of worksheet)

Once students have completed the worksheet, have them discuss Task 2 from it. Ask them why the function might "level off" in Task 2, no matter which part they completed. You might even want to introduce the concept of *carrying capacity*, which is the maximum number in a population (for Task 2a it is fish, for Task 2b it is toads) that the environment (the pond) can support. Since the tasks involve a pond, there are finite resources. A factor that students are likely to mention first that impacts the carrying capacity is the availability of food (such as aquatic vegetation, insects, or smaller fish); however, space and shelter (especially related to sunlight, temperature, and places to hide from predators) can also influence the carrying capacity. So, the rate of increase of the population, is more rapid at first and then slows down before it levels off. The *a* stretches the graph, while the *c* shifts the graph. Discuss how both impact the carrying capacity, where the functions levels off (for both Task 2a and Task 2b).

If students have completed both Tasks 2a and 2b, discuss what aspect of the equation for Task 2b would shift the graph to the right. Then consider how the physical scenario might explain this. Why might the toad population see an increase that lags about after the increase in fish? [Hint: Think about the fact that toads can eat baby fish that are much smaller than them. Also, insects eat fish larvae and eggs, and toads then eat the insects.]

These conversations are important since they reinforce that all of the algebraic and graphical manipulation students have done in previous lessons are related to real-world phenomena and how mathematical models are created from parent functions.