Lesson 4

TITLE OF LESSON: Applying Transformations to Any Function

ESTIMATED TIME FOR LESSON (IN MINUTES): 30 minutes

SUGGESTED FORMAT (check all that are appropriate):

Individual in-class
Collaborative in-class
Individual homework
Collaborative homework

OVERVIEW:

- Students use what they have learned in previous lessons to apply transformations to any given function.

PREREQUISITE IDEAS AND SKILLS:

- Students should understand what a transformation is and be able to apply multiple transformations (shifting, stretching, reflecting) to a quadratic function.

MATERIALS NEEDED TO CARRY OUT LESSON:

- Instructor access to Internet for instructor demonstrations
- Students access to Internet for students' collaborative exploration
- The "Example Formulations Key" from Lesson 3.
- The "Transformations Worksheet" and Answer Key

CONCEPTS TO BE LEARNED/APPLIED:

Guided exploration will help students to:

- Recognize horizontal shifts, vertical shifts, horizontal or vertical stretches or compressions, and horizontal or vertical reflections in any given function when written in a familiar formulation.
- Apply these same transformations to the graph of a "parent function."

ACTIVITIES:

ACTIVITY 1: Abstracting Our Formulation

Goals:

- Students will write down a general formulation and set of rules for applying transformations to any given function.

Steps:

Introduction

- Show students the quadratic “parent function”, $f(x) = x^2$.
- Below this, write your class formulation for a quadratic function as developed in the previous lesson.
- If your formulation contains the expression “$f(x)$”, replace it with “y” for clarity, explaining that we don’t want to use the same name ($f(x)$) to refer to two different things (the parent function and the transformed function).
- Quickly review the term “parent function” and the idea that a parent function gives us a shape, and our formulation (along with a set of rules) tells us how to change that shape using transformations.

Desmos Setup

- Have students open a new Desmos graph.
- In line one, enter the parent function, $f(x) = x^2$
- In line two, enter our quadratic formulation. Click “add sliders.”
- Now change the exponent, 2, in our parent function to the number 3, then ask students the following questions:
- What is the new shape of our parent function?
  (Answers may vary; any description is good. The new function is a cubic function.)
- How can we change our formulation so that the shape it gives matches that of the new parent function?
  (Change the exponent, 2, to the number 3.)
- Will our rules for applying transformations still work with the new parent function?
  (Answers may vary, but intuitively, yes, they should still work. We’re about to test that out!)

Applying Transformations to Various Functions

- Write any or all of the following parent functions on the board:
  $f(x) = x^3$, $f(x) = |x|$, $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$
- Ask students to try each parent function in line one, then alter the formulation in line two to match. Instruct them to play with the sliders to determine in each case whether they produce expected results. Specifically, does the slider that shifts the graph up and down still shift the graph up and down, etc. Give students several minutes to try this exercise. Students should be able to shift, stretch, and reflect any of the given parent functions by adjusting the formula in line 2.
- Ask the following questions:
- Were you able to use the formulation in line two to transform each given parent function?
  (If no, assist where needed. Students should be able to do this fairly easily.)
- In each case, did the sliders behave as expected?
  (If no, explore the reason a bit. Sliders should produce similar results to those produced for the quadratic function.)

- Now are you ready for a real challenge?
  (The class should shout “Yes!” as one, and may leap out of their seats in an emotional display of excitement. 😁)

Desmos Challenge

- Give students the following challenge: “Set up your formulation so that no matter what parent function you enter in line 1, you’ll be able to transform it with the sliders WITHOUT CHANGING LINE 2.”
- Hint: You are allowed to refer to “$f(x)$” in line 2 and Desmos will know what you mean.
- For possible solutions, see the bottom row on the “Example Formulations Key” from Lesson 3.
- Give high praise to the first few students who figure this out, then share the solution with the class. Have all students adjust line 2 to the general formulation using “$f(x)$” and encourage them to play with changing the parent function on line one to see what will happen.

In Conclusion

Congratulate your students and review with them (in your own words) what we’ve just accomplished:

- We started with no idea how to graph a quadratic function based on the numbers in the formula.
- We simplified the problem by removing the x term and discovered how to use parameters to shift, stretch, and reflect our basic parabola in the vertical direction.
- We incorporated two parameters into a specific formulation and developed a set of rules for using those parameters to graph any parabola centered on the y-axis.
- We added a new parameter to our formulation to shift our parabolas left and right.
- We added a second new parameter to express horizontal stretching and reflecting.
- Now we’ve taken the parabola out of our formulation, replacing it with $f(x)$ so that we’re able to apply vertical and horizontal shifting, stretching, and reflecting to *any given parent function.*
- Whew!

Exercises (Optional)

You may wish to have students practice transforming functions on the provided “Transformations Worksheet.” A grading key is also provided. Note that the formulation used for the transformed functions on the key may not match your own, but students’ answers should be algebraically equivalent, leading in each case to the same graph presented as the answer on the key.

What is left to do? (Optional)

- Can we graph any parabola given in standard form, such as $f(x) = 4x^2 + 3x - 2$?
  (Not directly; this does not fit our formulation!)
- Can we start with a standard-form quadratic and move things around so that we do know how to graph it?
  (Answers may vary; we can, but we may not yet know how.)
- One method for doing this is called "Completing the Square." We will see that in action in Lesson 5.
- Note to instructor: An MIP ARC on Completing the Square is also available.