Example Formulations Key

Follow each column down to see one way students' initial choice of formulation may evolve into a final formulation and set of rules. Note that students' choices may differ at each step; some examples of interesting choices are denoted by an asterisk in column C.

A	B	C

Possible formulations from **Lesson 2** along with rules for graphing:

$f(x) = ax^2 + k$	$f(x) = a(x^2 + k)$	$y - k = ax^2$
1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units.	1. Shift up k units. 2. If $a < 0$, reflect over x-axis. 3. Vertical stretch about the x-axis by a factor of a.	1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units.

These lead to possible formulations for **Lesson 3, Activity 1 (Horizontal Shift)**, with additional rules:

$f(x) = a(x - h)^2 + k$	$f(x) = a((x - h)^2 + k)$	$y - k = a(x - h)^2$
1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units. 4. Shift right h units.	1. Shift up k units. 2. If $a < 0$, reflect over x-axis. 3. Vertical stretch about the x-axis by a factor of a. 4. Shift right h units.	1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units. 4. Shift right h units.*

*Note that students may choose to add h rather than subtract, which would pair with the instruction to shift left rather than right h units.

These lead to possible formulations for **Lesson 3, Activity 2 (Horizontal Stretch/Shrink and Reflection),** with more additional rules:

$f(x) = a(bx - h)^2 + k$	$f(x) = a((bx - h)^2 + k)$	$y - k = a(b(x - h))^2$
1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units. 4. Shift right h units. 5. Horizontal compression by a factor of b about the y-axis. 6. If $b < 0$, reflect over y-axis.	1. Shift up k units. 2. If $a < 0$, reflect over x-axis. 3. Vertical stretch about the x-axis by a factor of a. 4. Shift right h units. 5. Horizontal compression by a factor of b about the y-axis. 6. If $b < 0$, reflect over y-axis.	1. Vertical stretch by a factor of a. 2. If $a < 0$, reflect over x-axis. 3. Shift up k units. 4. Horizontal compression by a factor of b. 5. If $b < 0$, reflect over y-axis. 6. Shift right h units.*

*Note that the choice to include inner parentheses pairs with placing horizontal compression and reflection *before* the horizontal shift rather than after.

These lead to the following general formulations for **Lesson 4**, applying the same set of rules in each case to graph any set of transformations applied to the parent function f(x):

$y = af(bx - h) + k$	$y = a(f(bx - h) + k)$	$y - k = af(b(x - h))$