Lesson 1

TITLE OF LESSON: Graphing Simple Quadratic Functions with Vertical Transformations

ESTIMATED TIME FOR LESSON (IN MINUTES): 50-75 minutes

SUGGESTED FORMAT (check all that are appropriate):

Individual in-class
Collaborative in-class
Individual homework
Collaborative homework

OVERVIEW:

- Students use an interactive Desmos applet to investigate how the a, b, and c parameter values in a standard-form quadratic function, $f(x) = ax^2 + bx + c$, affect its graph.
- Students investigate functions for which the value of b is zero and a is one and functions for which the value of b is zero and c is zero.
- Students draw conclusions about vertical shifts, stretches, and reflections.

PREREQUISITE IDEAS AND SKILLS:

- Students should be familiar with the Cartesian coordinate system.
- Students should understand what a function is and the different ways it can be represented (e.g., graphs, equations).
- Given an x-value, students should be able to calculate the corresponding y-value for a function and plot the resulting ordered pair.

MATERIALS NEEDED TO CARRY OUT LESSON:

- Instructor access to Internet for instructor demonstrations
- Student access to Internet for students' collaborative exploration
- "Transformations Worksheet" for Activity 2

CONCEPTS TO BE LEARNED/APPLIED:

Guided exploration will help students to:

- Understand that the shape of the graph of any quadratic equation is a parabola and identify which features of a parabola make each graph unique.
- Understand that the a, b and c parameters from the standard form of a quadratic do not directly correlate to certain identifying features of the graph, namely the vertex.
- Use shifting and stretching/reflecting to graph any quadratic function given in the form $f(x) = ax^2$ or $f(x) = x^2 + c$.

- Articulate a set of rules for interpreting the a and c values as vertical shifts, stretches, and reflections.

ACTIVITIES:

ACTIVITY 1: Introduction (20 minutes)

Goals:
- Students understand that all quadratics have a similar shape – a parabola.
- Students understand how specific parabolas differ from one another.
- Motivate the need to find a way of graphing parabolas which does not depend on choosing random points.

Steps:
Parabolas. This Desmos link (https://www.desmos.com/calculator/ckhkxgrgfk) shows several examples of quadratic functions written in standard form along with their graphs. Show students the graphs one at a time and lead a discussion about parabolas, posing the following questions with desired responses provided in parentheses:
- What do these graphs have in common?
  (Their shapes are similar; they all have two ends pointing in the same direction with one turning point, or vertex, in between.)
- What change(s) to these formulas would produce a graph that isn't a parabola? Have students suggest changes and try them out on the Desmos app.
  (The a-value would need to be zero, i.e. no x^2 term, or you would need a higher-powered term, or a non-polynomial function.)
- If I give you any random quadratic function, what do we know about the graph?
  (Its shape is a parabola. If students get more specific, such as "with a positive leading coefficient the graph points upwards," let them know we'll get to that, but it's not required at this point. What we're looking for here is what do ALL parabolas have in common?)
- What can we say about the graph of a specific quadratic function by looking at it?
  (We're looking for features that vary among different parabolas: location of the vertex, whether it opens up or down, how fat or skinny the graph is.)
- What will it take to successfully draw the graph of a specific quadratic function?
  (Our graph will need to show where the vertex lies, what direction it opens, and how fat or skinny it is.)

Graphing by Hand. Present students with a new quadratic function, $f(x) = 7x^2 + 5x - 4$, and ask them to graph it by plotting individual points. You may have them work in groups. Give them two minutes to get as far as they can, then lead the following class discussion:
- Does anyone have a graph showing the vertex?
  (Unlikely, but if anyone does, make sure it's accurate and that they got it by plotting points; you can praise them for finding it using other methods, and state that this proves

your point, which is that plotting individual ordered pairs is not a great method of graphing a function.)

- Collect ordered pairs from various groups and plot them where everyone can see. You may want to use this Desmos link (https://www.desmos.com/calculator/uqxqei1kay) to plot their individual points. Collectively, can you visually estimate where the vertex would be?
- How close did we get to creating a complete and accurate graph?
- What are the advantages and disadvantages to graphing by plotting points?
  (Possible advantages:
  You can always use this method if you don't know what else to do.
  Possible disadvantages:
  It isn't very accurate.
  It's labor-intensive and time-consuming.)

Desmos App. Instruct students to access the Desmos activity for this exercise (https://www.desmos.com/calculator/nokas4fy1s). Have them work independently or in pairs. Give them the following instructions:

- Use the sliders to change the values of the a, b, and c parameters for the red parabola. Note how the parabola moves on the screen in response to these changes.
- Try to get your red parabola to match each of the target parabolas that are in green, orange and blue and shown with dashed lines.
- You can see the required values for a, b, and c for each of the target parabolas by opening the corresponding folder, but that's cheating!
- Your effort is the important thing here, not your results. Do your best to match your parabolas, and then we'll talk about your observations.

Give them at least five minutes to do the best they can, then lead the following class discussion:

- The Desmos function we worked with has three changeable parameters, a, b, and c. What is a parameter? What happens when you change its value? (Changing any parameter value produces a new function with its own corresponding graph.)
- By changing a, b, and c in the formula, should you be able to graph any possible parabola?
  (Yes, if you were allowed to use any real number for a, b, or c.)
- Can you find the a, b, and c values needed to match the graph of any given particular parabola?
  (Answers may vary: this is difficult, but may be doable given enough trial and error.)
- Is there a relationship between the values of a, b, and c and the resulting graph? Is there a pattern here?
  (Again, answers may vary: there is a pattern, but it's quite difficult to discern. Students may come up with partial answers, such as "If a is negative, the graph points downward," but it is unlikely anyone will have a complete answer describing all the required features of a parabola listed earlier.)

- Given the formula for any parabola, we want to graph that parabola without plotting points. Is there a way to determine the vertex, direction, and girth of a parabola directly from a, b, and c?
  (Answers may vary: if students produce formulae based on a, b, and c (possibly from previous classes or a Google search), point out that this is not a "direct" way. Some instructors may interpret c as the y-intercept, b as the slope at the origin, and c as a vertical stretch factor, and students may attempt in their own words to describe what each of the three numbers "does" to the shape of the graph, but none of these will provide an easy and direct way of determining the vertex of the parabola.)

Conclusion. Conclude your introduction with a statement to this effect: "We're feeling that while a, b, and c do determine the features of a graph, we haven't found a simple, direct way to convert those values into a vertex so that we can easily sketch the graph by hand. That's because this is actually a very difficult problem. One way to solve a difficult problem is to solve a simpler version first. That's what we're going to do next by focusing on one parameter at a time."

ACTIVITY 2: Vertical Transformations (30 minutes)

Goals:

- Students will be able to graph by sight any quadratic function given in the form $f(x) = ax^2$ or $f(x) = x^2 + c$.
- Students will be able to explain exactly how to arrive at a correct graph based on the value of a, when b and c are zero, or the value of c, when $a = 1$ and b is zero.

Steps:

Graphing the parent function. Ask students what would be the simplest quadratic function we could graph? Take suggestions until you arrive at $f(x) = x^2$. Introduce the term "parent function." This is the parent function for all parabolas, since it is the simplest form of a parabola, and it displays the same characteristics that all parabolas have in common. Graph the parent function by plotting points. Point out features of the graph. State that students should know the shape of the parent function, and that we'll be graphing other quadratic functions by moving and changing that shape, similar to what we did with the Desmos app earlier.

Shifting, Stretching, Reflecting Introduction. Ask students to suggest simple ways to complicate the parent function formula. Take suggestions until you have at least the top three items on this list:

- Add a positive number.
- Subtract a positive number.
- Multiply by a number.
- Multiply by a negative number. (Optional - only list this one if it comes up.)

- Multiply by a fraction less than one. (Optional - only list this one if it comes up.)

If students suggest placing a number in parentheses with the x, such as $f(x) = (x + 1)^2$, put this idea on a separate list. Praise them for their creativity, but state that we are starting with simpler ideas for now and will work our way up to more complex functions later. You may clarify that you want to alter the parent function without the need for parentheses.

For each of the items on your list, ask students to predict what effect this will have on the graph of the function. For example, “If I add a positive number to $f(x) = x^2$, what do you think it will do to the graph of the parent function?”

Accept all predictions as valid, even if you know they are wrong. Allow students to use their own vocabulary. (The terms “Shifting,” “Stretching,” “Reflecting,” and “Transformation,” will be introduced on the worksheet.) Ask each responder whether they have a reason for thinking the way they are thinking, or are they making a wild guess? Listen but do not evaluate their reasoning.

Students’ reasoning may have to do with function inputs and outputs. If they have trouble making a prediction, encourage their thinking in this direction by suggesting they compare the output (or y-value) of the parent function to the output of the altered function for several different inputs (or x-values).

Once you’ve heard a few predictions about each item on the list, move on to the worksheet below.

Transformations Worksheet. Have students work individually or in groups. Instruct them to answer all the questions on the provided worksheet. Circulate among students as they work through the exercise to offer guidance.

Shifting, Stretching, Reflecting Discussion. With ten minutes of class time remaining, interrupt the exercise and lead a class discussion about the results. Some students will have gleaned all they need from the worksheet, and some may still be putting it together.

Test student understanding of the following vocabulary. Feel free to substitute in words that you or your textbook actually use, if they differ:

- Transformation
- Vertical Shift
- Vertical Stretch
- Vertical Shrink
- Reflection over the x-axis

Have students answer the following questions, using the above vocabulary as you discuss:

- What happens to the graph when you add a (positive) number to x^2? Why?
- What happens to the graph when you subtract a (positive) number from x^2? Why?
- What happens when you multiply x^2 by a positive number greater than one? Why?
- What happens when you multiply x^2 by a positive number less than one? Why?
- What happens when you multiply x^2 by a negative number? Why?

The "why?" answers may vary, but in general, these calculations (adding, subtracting or multiplying) change the y-value of each point on the graph, so it makes sense they should transform the entire graph in a similar way.

If students have difficulty expressing this, make a table comparing output values of the parent function to output values of an altered function for various different input values. For example:

	Parent Function	**Altered Function**
$\boldsymbol{x}$	$\boldsymbol{f(x) = x^2}$	$\boldsymbol{g(x) = 2x^2}$
1	1	2
2	4	8
3	9	18

From this the student may conclude that the graph of the parent function is stretched vertically because each output, or y-value, is doubled.