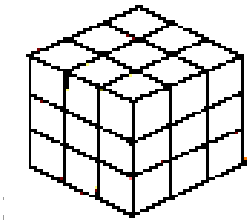
CUBE PAINTING

1. An 3cm x 3cm x 3cm cube is made from gluing together 27 small 1cm x 1cm x 1cm (unit) cubes. If this large cube is painted, allowed to dry and then broken apart into the small unit cubes, how many of them have paint on
   - no faces?
   - only 1 face?
   - two faces?
   - three faces?
   - four faces?

   Explain each answer.



Possible explanations given.

$f_0(3) = 1$
(there is only one small cube "inside" the big cube that doesn't have paint on it).

$f_1(3) = 6$
(each of the faces of the cube, and there are 6, has 1 small cube in the middle that has one face painted)

$f_2(3) = 12$
(each of the edges of the cube, and there are 12, has one small cube in the middle of the edge with two faces that are painted)

$f_3(3) = 8$
(the corner cubes, and there are 8, each have 3 faces painted)

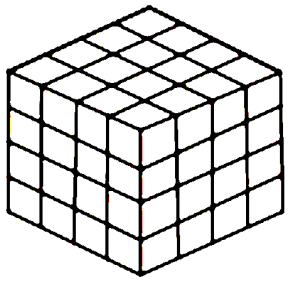
$f_4(3) = 0$
(no cubes will have 4 faces painted)

2. a) How many small unit cubes would you need to glue together to make a large 4cm x 4cm x 4cm cube?

b) If the outside of the large cube is painted, allowed to dry and then broken apart into the small unit cubes, how many of them have paint on

- no faces?
- only 1 face?
- two faces?
- three faces?
- four faces?

Explain each answer.



Possible explanations given.

$f_0(4) = 8$

(there is an interior cube "inside" the big cube that is 2 x 2 x 2 and none of the small cubes that make up the interior cube have paint on them; there are 8 of those small cubes).

$f_1(4) = 24$

(each of the faces of the cube, and there are 6, has a 2 x 2 square mades of 4 small cubes in the middle that each have one face painted)

$f_2(4) = 24$

(each of the edges of the cube, and there are 12, has two small cubes in the middle of the edge with two faces that are painted)

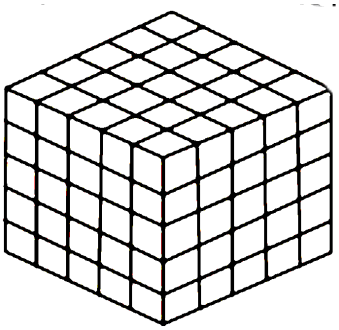
$f_3(4) = 8$

(the corner cubes, and there are 8, each have 3 faces painted)

$f_4(4) = 0$

(no cubes will have 4 faces painted)

3. a) How many small unit cubes would you need to glue together to make a large 5cm x 5cm x 5cm cube?
b) If this large cube is painted, allowed to dry and then broken apart into the small unit cubes, how many of them have paint on
   - no faces?
   - only 1 face?
   - two faces?
   - three faces?
   - four faces?



$f_0(5) = 27$

$f_1(5) = 54$

$f_2(5) = 36$

$f_3(5) = 8$

$f_4(5) = 0$

4. a) How many small unit cubes would you need to glue together to make a large 6cm x 6cm x 6cm cube? (Note that you are asked to do this without a visual image provided.)
b) If this large cube is painted, allowed to dry and then broken apart into the small unit cubes, how many of them have paint on
   - no faces?
   - only 1 face?
   - two faces?
   - three faces?
   - four faces?

$f_0(6) = 64$

$f_1(6) = 96$

$f_2(6) = 48$

$f_3(6) = 8$

$f_4(6) = 0$

5. Organize all the pertinent information for the cubes in questions 1 through 4 above in the following table.

Side Length of Big Cube	**Number of small cubes have paint on…**				
	No Faces $i = 0$	**1 Face** $i = 1$	**2 Faces** $i = 2$	**3 Faces** $i = 3$	**4 Faces** $i = 4$
3 cm $f_i(3)$	1	6	12	8	0
4 cm $f_i(4)$	8	24	24	8	0
5 cm $f_i(5)$	27	54	36	8	0
6 cm $f_i(6)$	64	96	48	8	0

6. a) How many small unit cubes would you need to glue together to make a large cube where each side measures n centimeters, in other words, to make an n cm x n cm x n cm cube?

n^3

b) If this large cube is painted, allowed to dry and then broken apart into the small unit cubes, how many of them have paint on no faces, only 1 face, two faces, three faces, and four faces? (Explain how you arrived at each answer.)

$f_0(n) = (n - 2)^3$

$f_1(n) = 6(n - 2)^2$

$f_2(n) = 12(n - 2)$

$f_3(n) = 8$

$f_4(n) = 0$

7) A constant function is a function whose outputs remain the same no matter the input that is used. A linear function is a function whose outputs increase at a constant rate for each unit increase in its inputs. The constant rate is called the *slope* of the linear function.

- Number of small cubes with no faces painted as a function of the side length of the big cube
- Number of small cubes with 1 face painted as a function of the side length of the big cube
- Number of small cubes with 2 faces painted as a function of the side length of the big cube
- Number of small cubes with 3 faces painted as a function of the side length of the big cube
- Number of small cubes with 4 faces painted as a function of the side length of the big cube

a) Which of the above would be a constant function? Explain your selection(s) in terms of the geometric context, the table given in #5, and the answers you came up with in #6.

f_3, and f_4

There are always 8 corners (with 3 faces painted), and never any small cubes with 4 faces painted.

b) Which of the above would be a linear function? Explain your selection(s) in terms of the geometric context, the table given in #5, and the answers you came up with in #6.

f_2

The edges are line-like and are the only small cubes that have only 2 faces painted.