Transformation Orders

https://www.desmos.com/calculator/frwmusfw1v

In Desmos (click here or copy the url above), you have a parent graph of $f(x) = \sqrt{x}$ graphed with a domain of [0,16] and a range of [0, 4]. The sliders on the left help create a transformation graph, $g(x) = af(bx - c) + d$. Move the sliders to answer the following questions.

(a) This question will be seeking to help you understand how the **order** of the vertical transformations matter by seeing where the point P goes.

(i) Let **d = 1**. What is the new point PT? Now, keep **d = 1** and let **a = 3**. What is the new point PT?

(ii) Reset your sliders (**a = 1** and **d = 0**). Now let **a = 3**. What is the new point PT? Now, keep **a = 3** and let **d = 1**. What is the new point PT?

(iii) In the previous two parts, PT should be the same point since **a = 3** and **d = 1** give us the same g(x). However, it is NOT true that we can apply transformations in any order we want. Desmos is accounting for the order for us. Since P has a y-value of 2 and our PT ends up with the y-value of of 7, which of the following gets us to 7 when we algebraically try to find PT?

(1) $(2 + 1) \cdot 3$

(2) $3 \cdot 2 + 1$

(iv) Which of the following corresponds with the arithmetic you chose?

(1) Applying **a = 3** and then **d = 1**

(2) Applying **d = 1** and then **a = 3**

(v) What order should be used when applying two vertical transformations?

(b) This question will be seeking to help you understand how the **order** of the horizontal transformations matter by seeing where the point P goes. If you haven't already, reset your sliders (**a = 1** and **d = 0**).

(i) Let **b = 3**. What is the new point PT? Now, keep **b = 3** and let **c = 2**. What is the new point PT?

(ii) Reset your sliders (**b = 1** and **c = 0**). Now let **c = 2**. What is the new point PT? Now, keep **c = 2** and let **b = 3**. What is the new point PT?

(iii) In the previous two parts, PT should be the same point since **c = 2** and **b = 3** give us the same g(x). However, it is NOT true that we can apply transformations in any order we want. Desmos is accounting for the order for us. Since P has a x-value of 4 and our PT ends up with the x-value of of 2, which of the following gets us to 2 when we algebraically try to find PT?

(1) $\frac{1}{3} \cdot 4 + 2$

(2) $(4 + 2) \cdot \frac{1}{3}$

(iv) Which of the following corresponds with the arithmetic you chose?

(1) Applying **b = 3** and then **c = 2**

(2) Applying **c = 2** and then **b = 3**

(v) What order should be used when applying two horizontal transformations?

(c) What values of **a** and **d** would make the range of $g(x)$ equal to $[-2, 10]$?

(d) What values of **b** and **c** would make the domain of $g(x)$ equal to $[-18, 14]$?

(e) Describe how the slider for **a** affects the graph of $g(x)$. What do you notice about the relationship between the value of **a** and the severity of the transformation?

(f) Repeat the previous questions with the sliders for **b**, **c**, and **d**.