Horizontal Transformations

https://www.desmos.com/calculator/6lta79xbov

In Desmos (click here or copy the url above), you have a parent graph of $f(x) = \sqrt{x}$ graphed with a domain of [0,16] and a range of [0, 4]. The sliders on the left help create a transformation graph, $g(x) = af(bx - c) + d$ where in this case $a = 1$ and $d = 0$. Move the sliders to answer the following questions.

(a) Which slider will shift the graph left and right? Which slider will horizontally stretch or compress the graph?

(b) For each of the following cases, find the equation for $g(x)$ and the domain for $g(x)$.

(i) Horizontally shift the graph right 3 units.

(ii) Horizontally shift the graph left 2 units.

(iii) Horizontally stretch the graph by a factor of 2.

(iv) Horizontally compress the graph by a factor of 3.

(c) Using the graphs of $f(x)$ and $g(x)$, and the labeling in Desmos, write down the points for R, P, and S on $f(x)$ and the points RT, PT, and ST on $g(x)$. Using these points, describe what happened to the x-values during the transformation. Similarly, describe what happened to the y-values. Use the placement of the 3 (or 2) in your equation to explain your response.

(i) Horizontally shift the graph right 3 units.

(ii) Horizontally shift the graph left 2 units.

(iii) Horizontally stretch the graph by a factor of 2.

(iv) Horizontally compress the graph by a factor of 3.

(v) Now, you control the x-values in the table for $g(x)$. For each of the above parts, find what x-values give you a table for $g(x)$ that has the same y-values as the $f(x)$ table. Compare the x-values in the $g(x)$ table with the x-values in the $f(x)$ table. What mathematical operations should be applied to the x-values in the $f(x)$ table to obtain the x-values in the $g(x)$ table?

(d) Suppose $c = 0$. What value of **b** would make the domain of $g(x)$ equal to $[0, 32]$?

(e) Suppose $b = 1$. What value of **c** would make the domain of $g(x)$ equal to $[-7, 9]$?

(f) From the above exercises, you can observe that the arithmetic for the transformations seems backwards (especially when compared with the vertical transformations). In this exercise, we are going to explore *why* this is the case.

(i) Consider $g(x) = f(x - 5)$. Write out what $g(2)$ is.

i. Notice that 2 is the input of $g(x)$, not $f(x)$. What is the input of $f(x)$ when 2 is the input of $g(x)$?

- *Note: here, we have transformed backwards (we went from an input of $g(x)$ to an input of $f(x)$).*

ii. Now let 2 be an input of $f(x)$. What is the input of $g(x)$ when 2 is the input of $f(x)$?

- *Note: here, we have transformed in the usual direction (we went from an input of $f(x)$ to an input of $g(x)$).*

iii. What is the input of $g(x)$ when 3 is the input of $f(x)$? What is the input of $g(x)$ when 4 is the input of $f(x)$? In general, what is the input of $g(x)$ when x_0 is *any* input of $f(x)$?

We should now have an idea as to <u>why</u> the arithmetic for the transformations seems backwards. The answer lies in <u>who</u> the x-value being transformed belongs to (i.e. which function's domain is it in?), and who the x-value in the algebraic expression of the function notation belongs to.