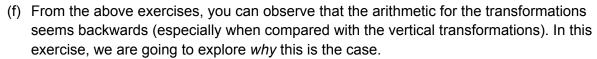
Horizontal Transformations

https://www.desmos.com/calculator/6lta79xbov

In Desmos (click <u>here</u> or copy the url above), you have a parent graph of $f(x) = \sqrt{x}$ graphed with a domain of [0,16] and a range of [0, 4]. The sliders on the left help create a transformation graph, g(x) = af(bx - c) + d where in this case a = 1 and d = 0. Move the sliders to answer the following questions.

- (a) Which slider will shift the graph left and right? Which slider will horizontally stretch or compress the graph?
- (b) For each of the following cases, find the equation for g(x) and the domain for g(x).
 - (i) Horizontally shift the graph right 3 units.
 - (ii) Horizontally shift the graph left 2 units.
 - (iii) Horizontally stretch the graph by a factor of 2.
 - (iv) Horizontally compress the graph by a factor of 3.
- (c) Using the graphs of f(x) and g(x), and the labeling in Desmos, write down the points for R, P, and S on f(x) and the points RT, PT, and ST on g(x). Using these points, describe what happened to the x-values during the transformation. Similarly, describe what happened to the y-values. Use the placement of the 3 (or 2) in your equation to explain your response.
 - (i) Horizontally shift the graph right 3 units.

	(ii)	Horizontally shift the graph left 2 units.
	(iii)	Horizontally stretch the graph by a factor of 2.
	(iv)	Horizontally compress the graph by a factor of 3.
	(v)	Now, you control the x -values in the table for $g(x)$. For each of the above parts, find what x -values give you a table for $g(x)$ that has the same y -values as the $f(x)$ table. Compare the x -values in the $g(x)$ table with the x -values in the $f(x)$ table. What mathematical operations should be applied to the x -values in the $f(x)$ table to obtain the $f(x)$ table?
(d) Suppose $c=0$. What value of b would make the domain of $g(x)$ equal to $[0,32]$?		
(e) Suppose $b=1$. What value of ${\bf c}$ would make the domain of $g(x)$ equal to $[-7,9]$?		



- (i) Consider g(x) = f(x 5). Write out what g(2) is.
 - i. Notice that 2 is the input of g(x), not f(x). What is the input of f(x) when 2 is the input of g(x)?
 - Note: here, we have transformed backwards (we went from an input of g(x) to an input of f(x)).
 - ii. Now let 2 be an input of f(x). What is the input of g(x) when 2 is the input of f(x)?
 - Note: here, we have transformed in the usual direction (we went from an input of f(x) to an input of g(x)).
 - iii. What is the input of g(x) when 3 is the input of f(x)? What is the input of g(x) when 4 is the input of f(x)? In general, what is the input of g(x) when x_0 is any input of f(x)?

We should now have an idea as to \underline{why} the arithmetic for the transformations seems backwards. The answer lies in \underline{who} the x-value being transformed belongs to (i.e. which function's domain is it in?), and who the x-value in the algebraic expression of the function notation belongs to.