Avoid Memorization: Deriving Identities

OVERVIEW: In this activity, students will use their critical thinking and algebraic skills to derive multiple trigonometric identities. They will understand how a few trigonometric identities and facts can be used to create many other identities, thereby reducing their need for memorization. Problem-solving skills are encouraged as students select an appropriate substitution for a variable in an equation or add a clever "0." Thus, this activity encourages students to be active participants in their own education as they deepen their understanding of trigonometry and develop a broader appreciation for the cohesive nature of mathematical principles and their applications.

MATERIALS NEEDED TO CARRY OUT THE LESSON:

- Reference List of Identities
- In-Class Activity Worksheet

CONCEPTS TO BE LEARNED/APPLIED:

- Students will understand that they do not need to memorize every mathematical fact (e.g. trigonometric identity) they encounter.
- Students will understand that they can derive many of the trigonometric identities using only the reciprocal and quotient identities, three additional trigonometric identities, and a characteristic (even/odd) of the sine and cosine functions.
- Students will understand the importance of being creative in mathematics as they discover the useful problem-solving skill of multiplying an expression by a clever "1" or adding a clever "0" to an expression.

INSTRUCTIONAL PLAN:

The instructor is encouraged to utilize this activity throughout the units that cover trigonometric identities, beginning with the Pythagorean Identities. For the first part of the activity, the instructor may provide a short lecture reminding students of the trigonometric reciprocal and quotient identities, as well as the meaning of even/odd functions and how that applies to the sine and cosine functions. The activity handout may be used as a set of interactive guided notes where the instructor lectures over the portions of the activity that the students are asked to read, and the students work in groups on the exploration portions of the handout. Or, students may read and work through the activity handout in groups and the instructor provides assistance as needed.

The instructor is encouraged to utilize this packet in multiple class periods and as the content is encountered in class. It is not intended for students to complete the packet all at once as this would likely be overwhelming! Each part of the activity begins on a new page and an instructor may opt to only provide the needed information (part/section) for a particular class period. At the end of the unit on identities, students will be able to compile their work into a complete packet for future reference. Moreover, as the need for identities

appears in future sections, the instructor is encouraged to recall and enact the ideas presented in this packet to reinforce this approach of developing formulas.

Lastly, after each part of the packet is covered in class, the instructor should consider having the students reflect on their work to emphasize that they only needed a small number of foundational identities as starting points and that they performed fairly basic algebraic operations to obtain the desired result. Directions for a reflection are provided below and can be used after each part of the activity is completed.

Write.a.reflection.detailing.your.work.and.thought.process.in.creating.the.identity.formula(s)¡.Use.the.following.prompts.as.a.guide¡

7¡ What.foundational.identities.did.you.use.as.starting.points?
8¡ What.was.the.desired.outcome–identity?
9¡ Describe.(in.words).the.connection.between.the.starting.points.and.the.desired.outcome¡
0¡ What.algebraic.operations.did.you.use.and.why.were.they.useful.to.obtain.the.desired.result?

MIP COMPONENTS OF INQUIRY:

Active Learning: In this activity, students are asked to create trigonometric identities by utilizing and enhancing their problem-solving skills. For example, students are asked to select an appropriate substitution for a variable in an equation or to create a clever form of the integer "0" that they can use to manipulate an expression. They perform the computations of simplifying the trigonometric expressions and evaluate their actions as they work towards deriving a trigonometric identity. This activity encourages students to explore and connect ideas, moving beyond rote memorization to a more profound comprehension of trigonometry. Moreover, this activity promotes active learning by transforming students from passive recipients of information into active participants in establishing trigonometric identities.

Meaningful Applications: Oftentimes, trigonometric identities are provided by the instructor and students simply use these identities via "plug-and-chug" type problems. Through student exploration in this activity, students will see the connection between many of the identities and how they interact with each other. As students identify and understand mathematical relationships within the trigonometric identities, they will develop and utilize mathematical strategies such as multiplying by a clever "1" or adding a clever "0," which is a strategy that is used throughout mathematics. Thus, students may deepen their understanding of trigonometry, as well as develop a broad appreciation for the cohesive nature of mathematical identities.

Academic Success Skills: This activity cultivates academic success skills by guiding students through an exploration of the derivation of multiple trigonometric identities. These tasks, some of which are more challenging than others, encourage students to use their critical thinking skills as they apply and use their previous knowledge to construct these identities. Through their discovery and reflection of the interconnectedness of the trigonometric identities, a growth mindset is fostered, which encourages students to persist.