Angle Sum Identities

In this activity, we will derive the angle sum identity for cosine:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

This identity (along with the angle sum identity for *sine*) will be extremely important for discovering *new* trig identities.

Part 1: GeoGebra Introduction (guided by your instructor)

GeoGebra Basics (guided by your instructor):

- Open GeoGebra and explore the basic tools for drawing lines, line segments, and angles.
- Practice constructing a triangle and identifying its centroid (or other centers, such as the circumcenter, orthocenter, or incenter)

Exploration Activity:

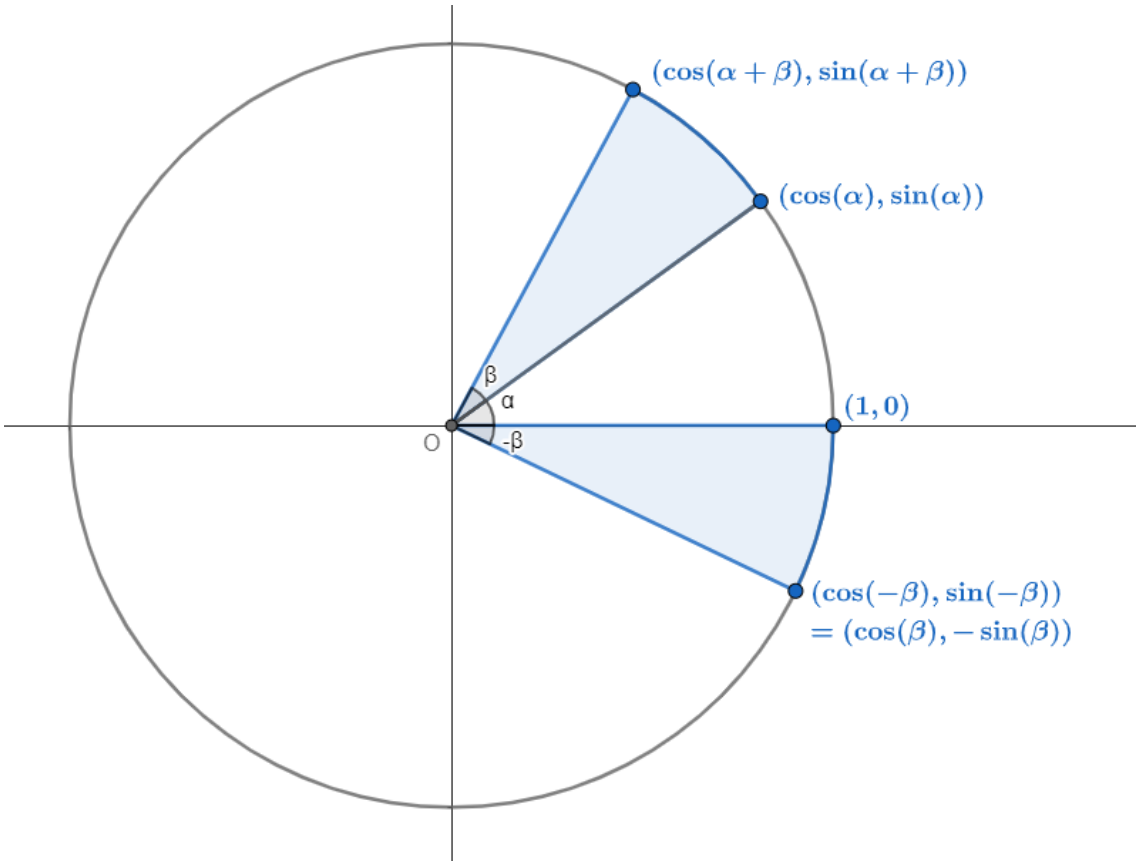
- Watch the one-minute YouTube video explaining why the angles in a triangle sum to 180 degrees.
- Reconstruct the diagrams shown in the video using GeoGebra.

Part 2: Exploring the Unit Circle

- Use GeoGebra to plot the following points in the unit circle (as shown in the diagram on the following page):

  $(1,0)$
  $(\cos(\alpha), \sin(\alpha))$
  $(\cos(\alpha + \beta), \sin(\alpha + \beta))$
  $(\cos(-\beta), \sin(-\beta))$

  You may assume that α and β are generic angles, and you should create these points as movable points (except for (1,0) of course).



This diagram can be found here.

- What do you notice about these 4 points? Is there anything special about pairing them up in a certain way (in terms of angles, distances, etc.)?

- Use the distance formula to express the distance between $(1,0)$ and $\big(\cos(\alpha+\beta), \sin(\alpha+\beta)\big)$.

- Use the distance formula to express the distance between $(\cos(\alpha), \sin(\alpha))$ and $(\cos(-\beta), \sin(-\beta))$.

- What can we say about these two distances? Use your answer to form an identity and simplify completely. Your final answer should give an identity for $\cos(\alpha + \beta)$.

- The identity you derived (i.e., $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$) is called the angle sum identity for cosine. Which trig identities did you use in your simplification?

- How could this identity be used to find the exact value of $\cos(75°)$? What about $\cos(105°)$? Do this below. [Hint: Is $75°$ the sum of two "nice" angles?]

Final answers:

$\cos(75°) =$ _____

$\cos(105°) =$ _____

Part 3: Further Exploration

How could we prove the angle sum identity for sine?

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$

Could we use a different diagram? Discuss your ideas and draw a possible diagram below.