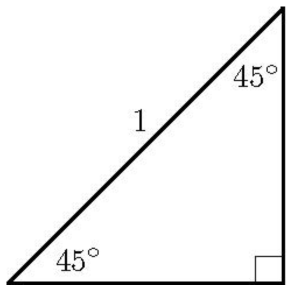
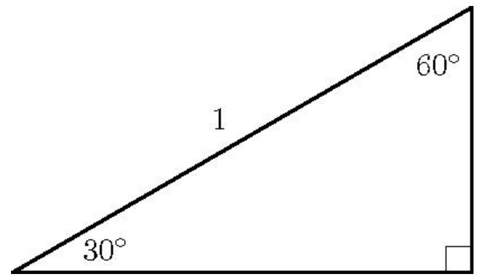
Exploring Special Triangles

In this activity, we will explore three "special" right triangles:

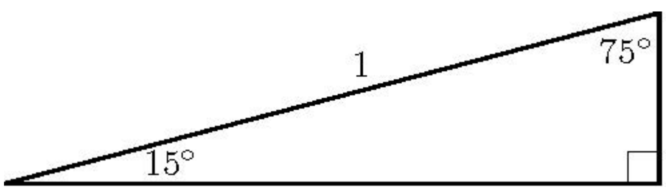
45-45-90:



30-60-90:



15-75-90:



Our goal is to find the missing side lengths. We assume the hypotenuse has length 1 in each case.

Estimation:

1. Estimate the lengths of the two legs of the **45-45-90 triangle** (see picture above). Hint: Is the bottom leg 50% as long as the hypotenuse? 60%? 75%? Make an estimate.

   Record your estimates for the lengths of the two legs (expressed as decimals):

   ___________ ___________

2. Now, estimate the lengths of the two legs of the **30-60-90 triangle** (see picture above):

   Record your estimates for the lengths of the two legs (expressed as decimals):

   ___________ ___________

3. Finally, estimate the lengths of the two legs of the **15-75-90 triangle** (see picture on previous page):

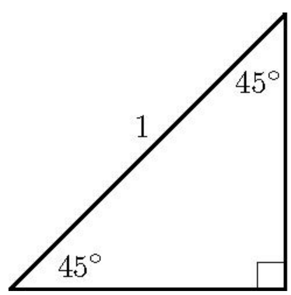
   Record your estimates for the lengths of the two legs (expressed as decimals):

   ____________   ____________

Derivation:

4. Now, find the exact lengths of the legs of the **45-45-90 triangle**.

   Hint: Label the sides. Think about symmetry and use any relevant theorems.



   Record your answers here (if there are square roots, rationalize the denominator so that we all have a uniform way of writing the answer):

   _____________   _______________

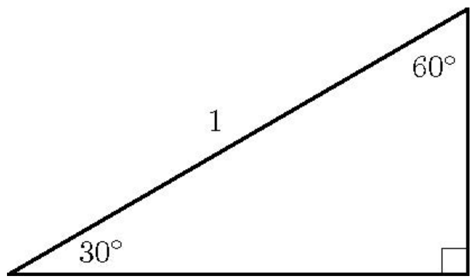
   Include decimal approximations (to 3 decimal places) for the two leg lengths found above:

   _____________   _______________

Compare your earlier estimates with the derived lengths. Discuss any discrepancies with your group.

5. Find the <u>exact</u> lengths of the legs of the **30-60-90 triangle**.

   <u>Hint</u>: Label the sides. Think about symmetry/reflection and use any relevant theorems. Can you make a larger triangle from two copies of this one? If you get stuck, ask your instructor for a hint.



Record your answers here (if there are square roots, rationalize the denominator so that we all have a uniform way of writing the answer):

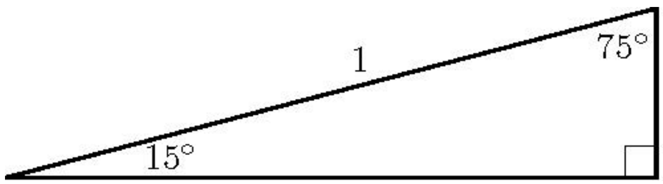
_____________ _______________

Include decimal approximations (to 3 decimal places) for the two leg lengths found above:

_____________ _______________

Compare your earlier estimates with the derived lengths. Discuss any discrepancies with your group.

6. Find the <u>exact</u> lengths of the legs of the **15-75-90 triangle**.



This problem is a bit more challenging! Try your best with your group to figure it out!

Record your answers here (if there are square roots, rationalize the denominator so that we all have a uniform way of writing the answer):

_____________ _______________

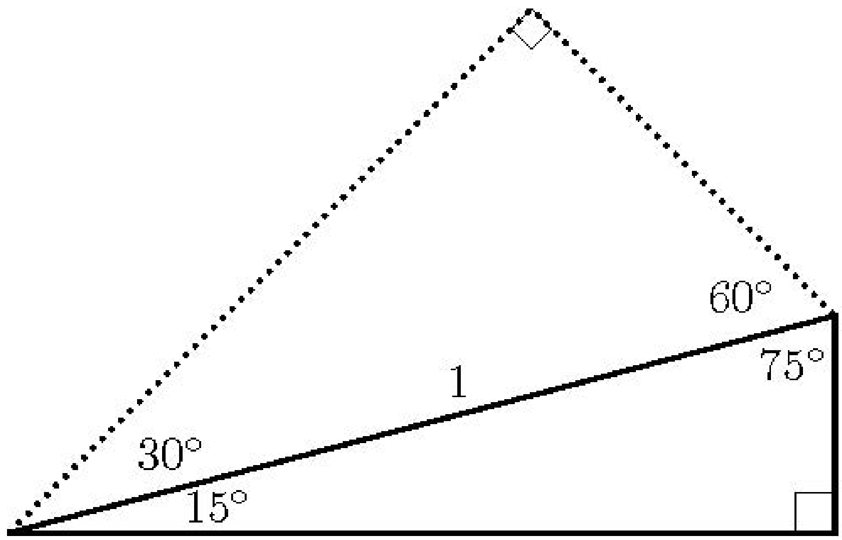
Include decimal approximations (to 3 decimal places) for the two leg lengths found above:

_____________ _______________

Compare your earlier estimates with the derived lengths. Discuss any discrepancies with your group.

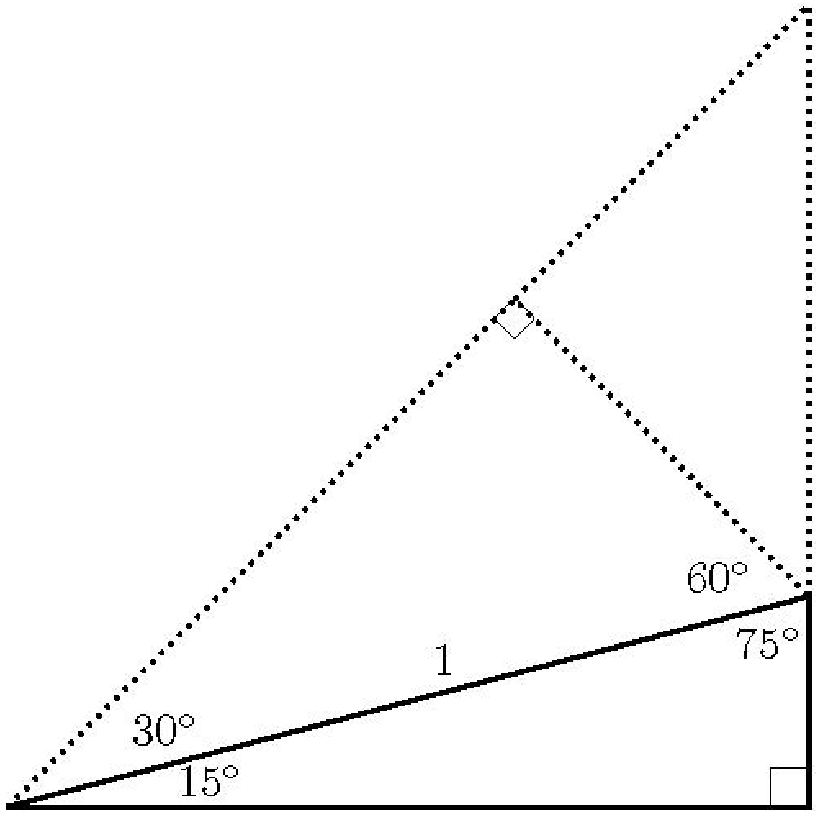
<u>Note:</u> This diagram should only be given to students <u>after</u> they have wrestled with the problem for a few minutes.

Hint #1



Note: This diagram should only be given to students if they are unable to make progress using "Hint 1" after several minutes.

Hint #2



Application Challenges:

(a) A ladder leans against a wall forming a 45° angle with the ground. If the ladder is 10 feet long, how high does the ladder reach up the wall? Round to 2 decimal places.

Solution: ________________

(b) A flagpole casts a shadow forming a 30° angle with the ground. If the length of the shadow is 52 feet, how tall is the flagpole? Round to 2 decimal places.

Solution: ________________

(c) An architect is designing a triangular window that forms a 30° angle at the peak. The base of the window is 6 feet long. What is the height of the window? Round to 2 decimal places.

Solution: ________________

Discuss the solutions to the application problems with your group and compare your approaches. Reflect on how the properties of the special triangles facilitated your problem-solving process.