LESSON TITLE: Exploring Special Triangles: 45-45-90, 30-60-90, and 15-75-90.

OVERVIEW

In this lesson, students will investigate the properties of three special right triangles: the 45-45-90 triangle, the 30-60-90 triangle, and the 15-75-90 triangle. The goal is to explore and derive the exact proportions of the side lengths of these triangles through student-centered inquiry. Students will begin by making conjectures about the side lengths based on geometric reasoning and estimation, and then verify their conjectures through various diagrams involving the triangles and then using the Pythagorean Theorem. Students also solve some application problems related to the three special triangles.

PREREQUISITE IDEAS AND SKILLS:

- Familiarity with the Pythagorean Theorem.
- Basic algebraic skills (solving quadratic equations).
- Understanding of square roots and simplification of radical expressions.
- Familiarity with the concept of congruence and basic geometric constructions.

MATERIALS NEEDED TO CARRY OUT THE LESSON:

- Activity worksheet with diagrams of the special triangles
- Ruler or measuring tape (optional)
- Protractors (optional)
- Graph paper (optional)
- Calculators (optional)
- Scissors for cutting triangle templates (optional)

CONCEPTS TO BE LEARNED / APPLIED:

- Students will understand and apply the specific ratios of side lengths for the 45-45-90, 30-60-90, and 15-75-90 triangles through exploration and verification using the Pythagorean Theorem.
- Students will develop problem-solving strategies by constructing and analyzing geometric figures to explore and confirm their properties.
- Students will enhance their critical thinking by making conjectures, testing their validity, and refining their understanding based on geometric principles.
- Students will solve application problems related to the three special triangles.

INSTRUCTIONAL PLAN:

The lesson begins with a brief review of the Pythagorean Theorem and its application in right triangles. Students will then be introduced to the three special triangles they will explore during the lesson: the 45-45-90 triangle, the 30-60-90 triangle, and the 15-75-90 triangle. The instructor will emphasize that students will explore and discover the exact side lengths relative to the hypotenuse, which is given as 1 in each case.

45-45-90 Triangle:

Students will receive a diagram of a 45-45-90 triangle with the hypotenuse labeled as 1. Working in small groups, they will estimate the lengths of the legs and record their estimates. They will be encouraged to use geometric intuition and (if the instructor chooses) measurement tools to support their estimates. Each group will then derive the exact lengths by applying the Pythagorean Theorem: $a^2 + a^2 = 1^2$, leading to $a = \frac{1}{\sqrt{2}}$ or $a = \frac{\sqrt{2}}{2}$. Afterward, they will compare their calculated lengths with their initial estimates and discuss any discrepancies, sharing their methods and findings with the class.

30-60-90 Triangle:

Next, students will examine a 30-60-90 triangle with the hypotenuse labeled as 1. They will again work in groups to estimate the side lengths and record their estimates. Instead of direct guidance, students will be provided with materials and hints (such as suggesting they explore reflective symmetry or the properties of equilateral triangles) to discover the relationship between the side lengths. They will use these hints to figure out that the side opposite the 30-degree angle is $\frac{1}{2}$ and then apply the Pythagorean Theorem to find that the other leg is $\frac{\sqrt{3}}{2}$. Groups will verify their results by comparing them with other groups and discussing the strategies they used to arrive at their solutions.

15-75-90 Triangle:

For the final part of the lesson, students will explore the lesser known 15-75-90 triangle. Initially, they will work in groups to estimate and attempt to derive the exact side lengths based on their understanding from the previous triangles. They will record their attempts and discuss their strategies. If no group can determine the side lengths, the instructor will provide a diagram as a hint. This special diagram is composed of two triangles (a 15-75-90 triangle and a 30-60-90 triangle) arranged in such a way that the long leg of the 15-75-90 triangle is the hypotenuse of the 30-60-90 one. Using this diagram, students can discover (based on what they learned about the previous two triangles) that the side lengths are $\frac{\sqrt{6}-\sqrt{2}}{4}$ for the shorter leg, and $\frac{\sqrt{6}+\sqrt{2}}{4}$ for the longer leg. Groups will then present their derivations and discuss the reasoning behind their approaches.

Important Note: It would be very helpful to ask students, "What happens if we <u>double</u> the lengths of all three sides of our 30-60-90 triangle? Would we still have a 30-60-90 triangle? What would the lengths of the sides be?" This would be helpful for students to understand that the leg lengths of a 30-60-90 triangle can be 1 and $\sqrt{3}$ (i.e., when the hypotenuse is 2). If would also reinforce the idea of "similar triangles." Similarly, if we multiply the side lengths of a 45-45-90 by $\sqrt{2}$ (not by 2!) then we obtain a 45-45-90 triangle that has hypotenuse $\sqrt{2}$ and legs of length 1.

Application Problems:
Students will investigate how these special triangles can be applied to solve other complex geometric problems. Each group will be given a set of challenging geometric problems where the use of special triangles can simplify the solution. They will present their solutions and explain how the properties of the special triangles were used in their reasoning.

ASSESSMENT

Students will be assessed based on their participation in group discussions, the accuracy of their calculations, and their ability to explain their reasoning. The completed worksheets and presentations will serve as a record of their findings and understanding.

MIP Components of Inquiry

Active Learning:
This activity engages students in active learning as they explore the properties of special right triangles. Rather than being given the side length ratios directly, students are tasked with making conjectures, conducting measurements, and applying the Pythagorean Theorem to verify their hypotheses. In the case of the 15-75-90 triangle, students are initially challenged to find the side lengths independently, fostering engagement and critical thinking. When provided with a “hint” diagram, they must interpret and apply this information to solve for the side lengths. This inquiry-based approach ensures that students are actively involved in discovering geometric principles through hands-on exploration and collaborative problem-solving.

Meaningful Applications:
This activity emphasizes meaningful applications by guiding students in identifying and understanding mathematical relationships within right triangles. As they work through the derivation of side lengths for the 45-45-90 and 30-60-90 triangles, students apply previously learned concepts such as the Pythagorean Theorem and properties of symmetry. The investigation of the 15-75-90 triangle requires them to use their knowledge of these simpler special triangles, promoting a deeper understanding of the geometric relationships. By extending their understanding to solve complex geometric problems, students recognize the practical utility of these triangles in various mathematical contexts, enhancing their problem-solving skills.

Academic Success Skills:
This activity cultivates academic success skills by empowering students to make mathematical discoveries through guided inquiry. The structured exploration encourages students to develop persistence as they navigate challenges, particularly in deriving the side lengths of the 15-75-90 triangle. The collaborative nature of the group work promotes communication and teamwork, as students discuss their findings and refine their approaches based on peer input. The process of estimating, calculating, and verifying side lengths enhances their mathematical reasoning, while the opportunity to present their solutions can help to build confidence and clarity in articulating mathematical concepts. This inquiry-based approach helps students build a strong foundation in geometric principles and reinforces their identities as capable problem-solvers.