Name: ________________________________

Instructional Activity: Why is the Pythagorean Theorem true?

Recall the statement of the Pythagorean Theorem:

> **The Pythagorean Theorem**
> In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In other words, if a right triangle has side lengths a, b, and hypotenuse c, then

$$a^2 + b^2 = c^2,$$

but why is this true? Where does this formula come from? We will investigate this question using ideas from geometry and algebra. In what follows, all of our computations will be in terms of a, b, and c.

Triangle/Square Activity:

1. Arrange the provided triangles into a large square.
   - Is there more than one way to do this?
     Yes, the triangles could be arranged into a solid square as well as a square with empty interior.
   - Can you arrange the triangles into a large square with a "missing" square in its interior?
     Yes, this arrangement will be key for what follows.
2. Place the provided square into the interior of the region created by your triangles.
3. Compute the area of the "filled" large square.
   Students will generally use (length)$\times$(width) for the computation of area here. In this case, the length and width will each be: $a + b$, giving an area of $(a + b)(a + b) = (a + b)^2$
4. Is there another method you could use to compute the area of the large square?
   The total area can be computed by summing together the areas of each of the 5 shapes (4 triangles and 1 square).
   - What shape can you make by combining two of the triangles?
     Two triangles can be combined to make a rectangle.
   - What is the area of this shape?
     This area has side lengths of a and b, hence its area is ab.
   - What is the area of one triangle?
     As the rectangle is comprised of two triangles, and the rectangle has area ab, it follows the area of one triangle is $\frac{1}{2}ab$.

   Summing the areas of the 4 triangles and 1 square gives: $4 \cdot \left(\frac{1}{2}ab\right) + c^2$, or equivalently, $2ab + c^2$.

5. Set the two different formulas you found for computing the area of the large square equal to each other.
   Setting the formulas equal gives $(a+b)^2 = 2ab + c^2$.

   - What happens if you simplify?
     Simplifying gives $a^2 + 2ab + b^2 = 2ab + c^2$, and subtracting $2ab$ from both sides yields the Pythagorean Theorem $a^2 + b^2 = c^2$!

This brings us to an important question – How do you know the shape you made in Step 1 (with side lengths $(a+b) \times (a+b)$) was actually a square?

6. Can you provide an example of a shape having equal side lengths that *is not* a square?
   A parallelogram (without having any right angles) can also have four equal side lengths.

7. What must the angles of the shape be to guarantee it is a square?
   A square must have four 90 degree angles.

As a follow-up, how do you know the provided shape (with side lengths $c \times c$) was a square?

8. Recreate the "filled" large square from Step 2.

9. What is the sum of the angles of a triangle?
   The sum of the angles must be 180 degrees.

10. Use what you know about complementary and supplementary angles to explain why the $c \times c$ shape must be a square.
    Each side of the "filled" square is comprised of three angles, one from each of the two triangles, along with one from the provided shape. These angles come together to form a straight line and hence must be supplementary (sum to 180 degrees). As we know the angles of a triangle must sum to 180 degrees (and one of those angles is already 90 degrees) it follows the other two angles are complementary. Using the notion of corresponding angles and transversals we know the two angles provided by the triangles along the square are complementary. Hence, it follows the angles of the $c \times c$ "shape" must be 90 degrees.