LESSON TITLE: Why is the Pythagorean Theorem true?

OVERVIEW: This lesson provides students a hands-on proof of why the Pythagorean Theorem is true. Students will use manipulatives to create a geometric shape whose area, when calculated using differing methods, establishes the theorem.

PREREQUISITE IDEAS AND SKILLS

- Familiarity with computing the area of geometric shapes (triangle/square)
- Understanding of why the angles in a triangle sum to 180 degrees
- Basic algebraic skills (multiplying binomials, manipulating equations)
- Understanding of complementary/supplementary angles

MATERIALS NEEDED TO CARRY OUT THE LESSON

- Activity worksheet
- Triangle / Square Manipulatives. (Click here for a GeoGebra diagram which can be printed and cut into 5 pieces for each student. The black dot on the bottom side can be dragged to change the shapes of the triangles.)

CONCEPTS TO BE LEARNED/APPLIED

- Students will explore and evaluate various configurations created by their manipulatives.
- Students will apply the formulas for the area of a triangle and square.
- Students will understand that not all shapes with four equal side lengths are squares, and will be able to determine which are.
- Students will apply the rule that the sum of the angles in a triangle equals 180 degrees.

STRUCTURE OF THE CONCEPTS TO BE LEARNED

- The Pythagorean Theorem states: "In a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides." Or equivalently, if a right triangle has side lengths a, b and hypotenuse c, then $a^2+b^2 = c^2$. Utilizing four manipulative triangles with these side lengths one can create a square with side length $a + b$ containing in its interior a square with side length c. Computing the area of the large square by: (1) computing length$\times$width, $(a + b)^2$; and (2) by summing the areas of the four triangles and interior square, $2ab + c^2$; and equating these formulas yields the theorem.

INSTRUCTIONAL PLAN

The activity begins by giving each of the students five manipulatives: four triangles (having side lengths a, b and hypotenuse c) and one square (having side length c). We note here that the students should not be told the final shape is indeed a square – it is simply a shape with four sides having equal length – the students will discover the shape is a square shortly.

The students will be asked to arrange the triangles into a square shape. There are many ways to accomplish this. Ultimately, we want the students to create a large square which is "missing" another square in its interior (this missing square will be the same size as the provided square). Students will then place the provided square into the empty region created by the triangles (filling the hole).

Now the students are asked to compute the area of the shape in front of them (the large square). Students will typically tend to use the area formula length×width. After the students have computed the area using this method, they are asked whether the area can be computed in another manner. Ideally, we want the students to now compute the area by summing the area of each manipulative in the large square.

With two separate formulas for the area of the large square in hand, the students are directed to equate them and simplify. Doing so establishes the Pythagorean Theorem!

Taking a step back, an important detail has been overlooked – was the shape created by the students, and the original manipulative which was provided, actually a square? The students realize each of the sides of the shape have an equal length, but that this does not mean the shape must be a square – it could be some other parallelogram.

As the created shape consisted of four right triangles at each corner, it is clearly a square; however, showing the providing manipulative is a square is a bit more involved. Students must show that the angles within the provided shape are 90 degrees. The large square the students created in their proof of the Pythagorean Theorem has sides which are comprised of three angles, one from each of the two triangles, and one from the provided "square". Since these angles come together to form a straight line the students know the angles must be supplementary. Since the angles of the triangles sum to 180 degrees (and one of those angles is already 90 degrees since it is a right triangle) the students know the other two angles are complementary. Using the notion of corresponding angles and transversals the students know the two angles provided by the triangles along the large square are complentary. Hence, the students conclude that the angle of the provided "square" must be 90 degrees; implying it is indeed a square.

MIP COMPONENTS OF INQUIRY

This section outlines how our activity will meet the Mathematical Inquiry Project (MIP) criteria for active learning, meaningful applications, and academic success skills.

<u>Active Learning</u>: This activity will engage students in active learning as they investigate the Pythagorean Theorem. Students will use manipulatives (provided paper cutouts of four triangles and one square) and select different approaches in an effort to establish the theorem. The triangles can be arranged in many different configurations, some of which will be helpful toward the end goal, while others will not. Students explore and evaluate each configuration they create to determine if it will be useful. Once a proper configuration has been found, students will compute the area of the resulting shape, which again can be completed in different ways. Students will discover that: (1) computing the total side length of the shape and using length×width; and (2) summing the areas of each individual piece within the shape, will give helpful, and equivalent, formulas for the area. Students will then realize that setting these formulas equal and simplifying will establish the Pythagorean Theorem.

<u>Meaningful Applications</u>: This activity emphasizes meaningful applications by guiding students to apply their understanding of the idea of "area" to set up and simplify an algebraic equation that

involves two different ways of algebraically describing the area of a certain geometric figure: (1) by multiplying the length of the figure by its width, and (2) by summing the areas of the individual components of the figure. The activity therefore encourages students to generalize across the differing contexts of geometry and algebra by connecting the geometric idea of area to algebraic expressions in a way that allows them to discover/prove the Pythagorean Theorem. This hands-on approach helps them justify claims about the areas involved and generalize their findings to understand why $a^2 + b^2 = c^2$.

Academic Success Skills: This activity will allow students to enhance their identities as learners by recognizing their capability to justify and explain important mathematical concepts themselves rather than simply being given formulas. In addition, students will realize how their prior knowledge can be applied in different situations to achieve new goals.