ACTIVITY 5: The One-Sample Mean T-Confidence Intervals

OVERVIEW:
In this activity, students will start by discussing with the instructor the limitations of the sample mean. The one-sample mean t-confidence interval will then be introduced as a more effective estimate of the population mean as it gives an idea of both accuracy and precision. Students will calculate 80%, 90%, 95%, and 99% confidence intervals for the three samples they selected in Activity 3 using technology, write their answers in a table, and answer questions about their confidence intervals to enhance understanding. An applet will be shown to illustrate the idea of a confidence interval and a discussion of what the confidence interval calculates will be discussed. The assumptions that need to be met to use the one-sample mean t-confidence interval will be presented and the students will determine whether the assumptions were met for the confidence intervals they created.

MATERIALS NEEDED TO CARRY OUT THE LESSON:
- In-Class Activity Worksheet
- Laptop with Excel and access to the internet

CONCEPTS TO BE LEARNED/APPLIED:
Students will understand the calculation and interpretation of confidence intervals for estimation.
- Students will understand why confidence intervals give them more information about the population in comparison to a sample mean.
- Students will understand that as the sample size increases, the width of the confidence interval decreases.
- Students will understand how there is a tradeoff between accuracy and precision when calculating confidence intervals.
- Students will understand that calculating a confidence interval is an attempt to find an interval that captures the population parameter being estimated with a certain level of confidence.
- Students will understand that to utilize the confidence interval formula presented, the data needs to be selected using a simple random sample and the distribution of the data needs to be approximately normally distributed for smaller sample sizes.
- Students will understand how to determine whether the assumptions are met for the confidence interval by reading how the data was selected and evaluating the normality of the distribution through a histogram and boxplot.

PREREQUISITE CONTENT KNOWLEDGE:

- Students need to know how to interpret histograms.
- Students need to know how to identify an approximately normal distribution based on a histogram.

INSTRUCTIONAL PLAN:

Present the following information to the students.

In Activity 3, the student groups calculated a sample mean. Unfortunately, sample means have limitations. When presented with a sample mean, it becomes challenging to determine its accuracy or precision as an estimate of the population mean.

A confidence interval, like the sample mean, is calculated from the sample data and provides an interval estimate of the population parameter. A confidence interval includes:

1. an interval computed from the sample that provides a measure of the variability or precisions. A wider interval shows more variability and less precision whereas a more narrow interval shows less variability and more precision.
2. a confidence level provides a measure of how likely it is that the interval includes the population mean that is being estimated.

Remember that like the sample mean, a one sample t-confidence interval is used to estimate a population mean.

The formula for the One Sample T-confidence interval is:

$$\bar{x} - E < \mu < \bar{x} + E$$

Point out to the students that they are taking the sample mean $\bar{x}$ and subtracting E (the margin of error) to form the lower bound of the confidence interval and then taking the sample mean and adding E to form the upper bound of the confidence interval.

Because the confidence interval is calculating an estimate for the population mean, the correct wording when stating the confidence interval is:

There is _____% confidence that the interval (_____, _____) captures the population mean……..

Once you have gone over the above, the students can work on the first page of the activity.

When the students are finished with the first nine questions of the Worksheet, show them the following applet:

https://digitalfirst.bfwpub.com/stats_applet/stats_applet_4_ci.html

Start with an 80% confidence interval and generate 25 samples. Explain to them that the applet is randomly selecting 25 simple random samples and calculating 80% confidence intervals from those samples. Of those confidence intervals, around 20 (80%) of them will capture the population mean and 5 (20%) will not. Then show them as you move the slider and increase the confidence level, the confidence interval gets wider, and more of the intervals capture the population mean.

When you are finished going over the applet, present the following material:

The confidence interval formula that was presented cannot be found for all data sets. Two assumptions need to be met before using these formulas.

Assumptions for One Sample T-Confidence interval:

- The units or subjects need to be selected using a simple random sample.
  If you are using a biased non-random sample, the formulas will yield biased results.
- The distribution of the data needs to be approximately normally distributed. The smaller the sample size, the more normal the data needs to be. Typically, if the sample size is 30 or more, it is fine to use the T-procedure even when the data is not normally distributed or there are non-extreme outliers.

Remind them that distributions that are approximately unimodal and symmetric are considered normal.

Unimodal means that there is just one distinct peak and symmetric means that there is approximately the same amount of data on the left of the peak to the right of the peak.

The second part of this activity allows them to evaluate the confidence intervals for the data they collected in Activity 2 and to determine whether the assumptions are met.

Note: The students were asked to save a copy of a spreadsheet with this data in Activity 3.