TITLE OF LESSON: Developing an Algebraic Test for Even and Odd Functions

ESTIMATED TIME FOR LESSON (IN MINUTES): 45 minutes

SUGGESTED FORMAT (check all that are appropriate):

- ☐ Individual in-class
- ☑ Collaborative in-class
- ☐ Individual homework
- ☐ Collaborative homework

OVERVIEW:
Students manipulate Desmos applets, make written observations, and use their observations to develop an algebraic test to determine whether a given function is even, odd, or neither.

PREREQUISITE IDEAS AND SKILLS:

- Definition of a function; Vertical line test
- Ability to evaluate and simplify functions using algebraic inputs, i.e. f(-x) = ?
- Familiarity with the Cartesian Coordinate System
- A strong sense of the concepts of even and odd symmetry of functions from a graphical perspective. (The "Categorizing Shapes and Functions by Symmetry" lesson plan can provide this if needed.)

MATERIALS NEEDED TO CARRY OUT LESSON:

- Copies of the provided worksheet for each group: Developing an Algebraic Test

CONCEPTS TO BE LEARNED/APPLIED:

- Types of symmetry
- Even and odd functions
- Algebraic tests for even and odd functions

INSTRUCTIONAL PLAN:

Goals

We want students to connect the graphical representations of even and odd symmetry with the algebraic meaning of even and odd. We want them to do this by moving from comparing individual ordered pairs to using algebraic representations that stand in for all ordered pairs.

Usage and Timing

This lesson is designed to be modular and flexible. You may use this as a stand-alone lesson or combine it with other lessons in the same ARC. Parts of this lesson may also be used independently. You may wish to give students more or less than the suggested amount of time for each activity depending on how thoroughly you wish to explore the topic.

Preparation (before class)

Print copies of the provided worksheet for each class member. Share a link to the provided applets, or similar applets in your preferred format.
Odd Functions: https://www.desmos.com/calculator/0rfczha2oz
Even Functions: https://www.desmos.com/calculator/sbew24npbo

(Note that the odd functions applet could be used with even functions, but not the other way around.)

Activity and Discussion

Introduction (10 minutes)

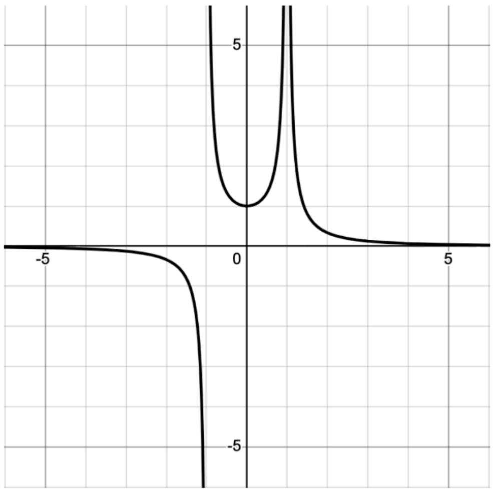
- (quick review) We can tell if a function is even or odd by looking at its graph. Show some examples and have students identify whether each is even, odd, or neither. Refer to the symmetry worksheet from the previous lesson for examples you can use here.
- Review the features of a graph that make it even or odd, as needed.
- Hand out the Developing Algebraic Tests Worksheet and have students look over the function formulas on page 1.
- Pose this question: Can we determine whether a function is even or odd by looking only at its formula? Gather student input on this. What is their confidence level that this is possible and/or that they will be able to do it? Can they make any guesses about what algebraic features may be telling?
- Show the applet and briefly demonstrate how the numbers can be manipulated by dragging points.
- Announce that our goal now is to come up with an algebraic way (a method that doesn't involve graphing) to determine whether a function is even or odd.

Group Work (15 minutes)

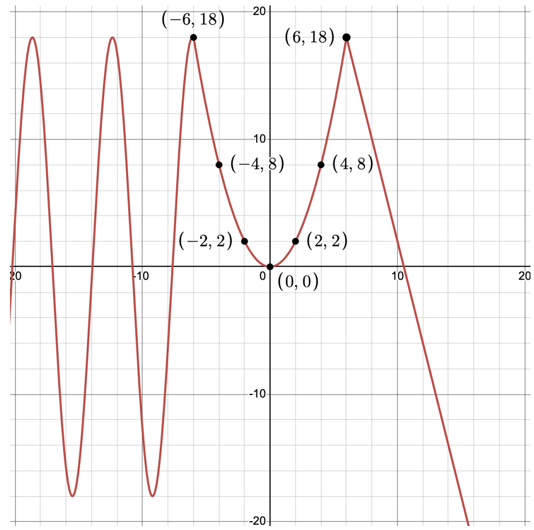
- Ask students to open the applet and use it to fill out as many tables as they can on the worksheet. Have students work in small groups. Encourage each group to try at least 4 different functions, 2 "easy" and 2 "hard" examples.
- Circulate and help as needed. Encourage different groups to choose different examples so you can collectively cover as many example functions as possible.
- Students may try to claim a function is even because of a few particular points, for example $f(2) = f(-2)$ and $f(-10) = f(10)$. Regardless of how many individual points show symmetry, the overall function may not be even. Use the examples below to illustrate.

- Example A is even on the interval (-1,1), but odd when you consider the domain (-∞, -1) ∪ (1, ∞). On the entire domain, the function is neither even nor odd, a fact that could easily be missed if you were to choose only certain points for testing.

Example A:



Example B:



- Example B shows that even when several plotted points appear to support the "even-ness" of a function, unexpected end behavior can render the entire function without symmetry. A similar example can be constructed from points plotted in any of the tables on the worksheet. Use this to convince students that a finite table of values alone will never show beyond doubt that a function is even or odd, thus justifying the need for an algebraic test, where x can represent all values in the domain at once.
- Use as much time as you have available to allow students to play with the functions. Be sure to leave time for the discussion that follows.
- When you need to move on, give students one more minute to skip to question #9 at the end of the worksheet. Have them write down their best guesses. Right or wrong, each guess will make a valuable contribution to the discussion to follow. You may want to give the hint to pay close attention to their table headings and also to the labeled points in the Desmos app.

<u>Follow-Up Discussion (20 minutes)</u>

This discussion should assist students in evaluating their results from the worksheet, with the goal of collectively arriving at one or more algebraic test(s) for even/odd functions. Classes will vary, so stay flexible in leading class discovery. Use these bullet points as a loose guideline.

- Have groups share their proposed tests for symmetry, with explanations.
- Collect results on the board or projector where everyone can see.
- If students aren't sure how to formulate a test, help translate their words into algebra.
- The more variety you collect in student responses the better; praise every effort as a great conjecture from which we have a chance to learn. (Even those which you know won't work.)

- Choose at least one proposed test to try out in front of the class. (If you see that a test *won't* work, that's a good one to start with!) Apply the test in question to both a polynomial function and a non-polynomial function from page 1 of the worksheet. Use Desmos to graph each function, demonstrating visually that it is even, odd, or neither. Try the algebraic test in question to see whether it yields the same result.
- Discuss the outcome. If the test fails to work, we know it's not a usable test. However if the test works in this case, does this mean we can depend on it to work in every case? (No!) How can we trust that our test will always correctly determine the even-ness or odd-ness of a function? How does the algebraic test reflect what we see in the graphs of even and odd functions? How does it turn on the definition of even and odd? Can we use some logic to reasonably conclude that the test will always work?
- Choose a non-polynomial function from the list on page 1 of the worksheet. Ask each group to individually try out their own proposed test (or to choose a test from the list you've compiled, if they didn't have one of their own) on the chosen function. Take a poll to see which tests gave the correct answer, and which did not.
- Examine any tests which failed to work, to determine as a class, why it failed. For example, a student may propose that: "If all the exponents are even, the function is even." (noting that a constant term has an exponent of zero, which is even). This is valid for polynomial functions (and we can use it for these cases), but applying it to a non-polynomial even function can show that a more general test is also required.
- Come to a consensus about which test works best, and write down an official test or list of tests for your class. You may need to push your students to modify their tests based on any incorrect results they're getting. Be sure the consensus they reach is for a test that actually works!
- Be open to and include correct variations, for example $-f(-x) = f(x)$ is the same as saying $f(-x) = -f(x)$.

Answer

An algebraic test that works well is this:
Compare $f(x)$ with $f(-x)$, simplified.
If $f(x) = f(-x)$, the function is even.
If $f(x) = -f(-x)$, the function is odd.
If neither of the above holds, the function is neither even nor odd.

Student answers should vary. Use their particular wording where you can, as long as it's clear and correct. Go to them for corrections as needed. This will help bolster their confidence and help them start to see themselves as competent mathematical scholars.