**Rationale for selecting/designing this problem/task sequence:**

* In this activity, students will use Desmos to investigate transformations of functions (vertical and horizontal translations, vertical stretches and compressions, and reflections). The goal is for students to not merely understand how to apply the various transformations, but to understand *why* those transformations work the way they do. The activity uses sliders in a way that allows students to see the effects of transformations quickly, while also using questions that lead students to discover the reasons the changes to the graphs occur.
* The activity also uses piecewise functions in several places, which is a bit unusual for an activity on transformations. The rationale for the use of piecewise functions is that many of the basic functions have various properties (symmetry, etc.) that make it difficult to determine which transformation is involved. For example, the function $f(x)=3\sqrt{x}$ can be thought of as a vertical stretch (by a factor of 3) or as a horizontal compression (by a factor of 9) because $f(x)=\sqrt{9x}).$ The same thing occurs for the basic functions $x^{2}, |x|$, $x^{3}$, $1/x$, etc. Similarly, several of the basic functions are either even (such as $x^{2}$ and $|x|$) or odd (such as $x^{3}$ and $1/x$) which can make reflections across the $x$- or $y$-axis problematic for student understanding. For example, for the function $f(x)=x^{3}$, a reflection across the $x$-axis is the same as a reflection across the $y$-axis. For this reason, we have used piecewise functions to more clearly demonstrate certain transformations (namely reflections and stretches/compressions).

**Prerequisite Knowledge:**

* Students should know that the graph of a function $f$ consists of the set of all ordered pairs of the form $(x,f(x))$.
* Students should know the graphs of basic functions, including $x^{2}, |x|, 1/x, \sqrt{x}, x^{3}$, etc.
* Students should know how to evaluate a function at a given input value.
* Students should know about composition of functions.
* Students should have some familiarity with using Desmos.

**Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix):**

The following objectives align with SLO #9 from the CEP matrix: “Identify and sketch graphs of functions including linear, polynomial, absolute value, rational, radical, piecewise functions, exponential, logarithmic, and use transformations of basic graphs.”

* Students will be able to define and describe the different types of transformations: translations, stretches and compressions, and reflections.
* Students will identify and perform vertical and horizontal translations of graphs, recognizing how adding or subtracting a constant from the variable $x$ shifts the graph left or right, and how adding or subtracting a constant to the function’s formula shifts the graph up or down.
* Students will identify and perform vertical stretches and compressions to graphs by multiplying the function by a constant factor greater or less than 1, respectively.
* Students will identify and perform reflections across both the $x$-axis and the $y$-axis, understanding that reflections across the $x$-axis occur when multiplying the function by -1 and that reflections across the $y$-axis occur when negating the variable $x$.
* Students will apply multiple transformations to a single graph and predict the cumulative effect of these transformations on the original graph, while understanding that the sequence (i.e. the order) of these transformations is important.
* Students will understand the reasoning behind why the various transformations work the way they do.

**Targeted concept to be developed:**

* In Activity 2, students will develop fluency with function transformations, focusing on how algebraic manipulations such as shifts, reflections, and stretches are visually represented in the graph of a function. They will learn to predict and explain the graphical outcomes based on changes in the formula of the function, gaining insight into why these transformations occur and how they are applied systematically across various functions (both basic functions, and functions in general). The activity strengthens their ability to logically connect algebraic expressions with their graphical representations.

**MIP Components of Inquiry:** **This section outlines how our activity will meet the Mathematical Inquiry Project (MIP) criteria for active learning, meaningful applications, and academic success skills.**

**Active Learning:** Students engage in active learning when they work to resolve a problematic situation whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned.

* In several Desmos slides in the activity, students use sliders to move various objects (graphs of functions, ordered pairs on graphs, etc.). This allows students to change the graphs based on how they select the inputs for the sliders. For example, in Slide 3, students select various values for a slider for $h$. They discover that increasing the value of $h$ in the function $f(x)=(x-h)^{2}$ results in a horizontal shift to the right. In the following slide, they then evaluate why the function shifts to the right (rather than to the left, as might be expected) by considering the input value for $x$ that gives the vertex of the parabola.
* Students evaluate their actions in several of the slides. For example, in Slide 2, students evaluate the actions from Slide 1 (which involve sliders that change the height of a function and an ordered pair on that function) by explaining why a vertical translation results from changing the value of $k$ in the function $f(x)=x^{2}+k$.
* In Slides 7 through 9, students perform various transformations of graphs by creating formulas. For example, in Slide 8, students find a formula for a function $f$ which translates the function $p(x)=|x|$ to the left 2 units and down 6 units. Students evaluate their answer by graphing the function in Desmos, and checking that the vertex is $(-2,-6)$. Similarly, students perform transformations in Slides 13 and 14, but this time with vertical stretches and compressions instead of vertical and horizontal translations.

**Meaningful Applications:** Applications are meaningfully incorporated in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure.

* Students use their prior knowledge of graphing functions (based on tables and ordered pairs) and their knowledge of graphs of basic functions to implement the new ideas of transformations of graphs.
* Students make and justify claims in, for example, Slide 3, where students make claims about horizontal shifts occurring when a value $h$ is added or subtracted to the input variable and justify those claims through tracking the vertex of a parabola.
* In various slides (for example, Slides 20, 25, 26, and 27) the fact that the order of transformations matters is an application of ideas from the composition of functions (which the activity assumes is prerequisite knowledge). For example, if $p$ is a parent function, and $h(x)=-x$, then functions $p(h(x))=p(-x)$ and $h(p(x))=-p(x)$

are not the same function. That is, the transformations are really compositions of functions, and the fact that the order of composition matters is the reason the order of transformations matters. Although this is not directly stated in the activity, the underlying ideas are there. Similarly, students see in Slide 20 that although the functions $f(x)=5|x|+2$ and $g(x)=5(|x|+2)$ both involve a vertical stretch by a factor of 5 and a vertical shift up by 2, they are not the same function, because the order of those transformations is different (i.e., the order of composition of functions is different).

**Academic Success Skills:** Academic success skills foster students’ construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community.

* This activity will allow students to make discoveries for themselves (about transformations of functions) rather than simply being told rules about these transformations. This promotes empowerment of students as they see themselves as someone capable of making their own mathematical discoveries and predictions and not simply someone who can memorize formulas and rules.

**Unit 2 Homework Activity: Transformations of Functions**

**Activity Instructions:**

The activity is designed for students to be able to complete on their own at home. The only thing they need is access to the internet and the following link to the Desmos activity:

<https://teacher.desmos.com/activitybuilder/custom/651ad5571b206fecdfedda3b>

Students will make their way through the 27 slides, answering questions along the way. The activity is self-explanatory, but we recommend that instructors try the activity so they can answer any potential questions students may have.

To utilize this activity in desmos, the instructor would need to create an account (can simply be linked to a google email address), create a class, and then assign the activity to the students. The instructor would need to provide a link based on their individual classes in Desmos.