**Rationale for selecting/designing this problem/task sequence:**

* This activity focuses on characteristics of graphs such as determining if a graph is a function, domain and range, function evaluation, intercepts, intervals of increase/decrease/constant, and piecewise functions. The two provided scenarios/activities are applications of piecewise functions, but problems with a low entry point. This allows students to tie together the ideas covered in College Algebra with functions and their graphs.

**Prerequisite Knowledge:**

* Students should have a basic understanding of functions (and associated characteristics).
* Students should be able to use and interpret function notation.
* Students should be able to write intervals in interval notation.
* Students should be able to identify intercepts of a graph.
* Students should understand basic properties of continuous functions.
* Students should be able to compare levels of increase or decrease by evaluating the rate of change.

**Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix):**

* Identify quantities and changes in quantities in mathematical representations, and distinguish constants from variables.
* Interpret functions and convert between their representations, including symbols, tables, graphs, and words.
* Perform operations on functions and identify the properties and characteristics of functions. Such properties and characteristics include domain and range, increasing and decreasing, one-to-one, inverses, even and odd, end behavior, relative extrema, and vertical and horizontal asymptotes.

**Targeted concept to be developed:**

* In Activity 1, students will develop their understanding of graphs, focusing on analyzing intercepts, intervals where the function is increasing/decreasing/constant, and examining domain and range in real-world scenarios. They will differentiate between independent and dependent variables and develop the ability to translate graphical data into mathematical representations and narratives. The activity aims to enhance students' understanding of functions and their applications, emphasizing function notation and the analytical skills necessary to interpret graphs.

**MIP Components of Inquiry:** **This section outlines how our activity will meet the Mathematical Inquiry Project (MIP) criteria for active learning, meaningful applications, and academic success skills.**

**Active Learning:** Students engage in active learning when they work to resolve a problematic situation whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned.

* In part one, students look at a graph and describe it with words. They select the appropriate features of the graph to point out. They can evaluate the validity of their story created for the graph when they see the second version of the graph with labeled tick marks.
* When provided with the second version of the graph, students identify specific characteristics of the graph (intercepts, intervals of increase/decrease/constant, domain/range).
* In part two, students again discuss key characteristics of a graph, but select which ones to include in the first question.
* Students perform an analysis of levels of increase by considering how steeply the graph is rising and supporting it with mathematics.

**Meaningful Applications:** Applications are meaningfully incorporated in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure.

* In this activity, students are provided two real-life situations to analyze. The students must take the information from sample graphs in class and be able to apply it to the new situations. This should assist students in generalizing how key characteristics of graphs/functions can be applied to basic graphs (such as lines/quadratics) as well as the functions given in this activity.

**Academic Success Skills:** Academic success skills foster students’ construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community.

* In this activity, students must struggle to apply key vocabulary/ideas about functions to new scenarios. This can help them realize that these key ideas are not arbitrarily discussed in math class alone.
* Students are allowed to be creative in the first problem by writing their own scenario that describes the graph. This gives them an opportunity to connect the graph with something they understand from real-life.

**Unit 1 Homework Activity:**

**Part One:**

1) Make up a story with the given graph. Be sure to interpret the intercepts and discuss what is happening when the graph is increasing, decreasing, or constant.

The graph starts at a positive depth. It increases at a constant rate, then increases more rapidly for a brief time. The depth remains the same for a while. Then the graph decreases to the axis. It appears that the $y$-intercept and $x$-intercept are positive values, but, without labels, it is unclear.

This could be a diver starting at a certain depth for scuba diving. Then going down slowly, then briefly more rapidly. The scuba diver could then stop and look at the coral reef (or a water creature, etc.) and then rise back up to the surface. Other options may be entering a cave, the level of grain in a grain silo, water in a pool, etc.

What is the independent variable? What is the dependent variable?

Time is the independent variable and depth is the dependent variable.



2) Consider the new version to the right.

\*Note: The solutions are with approximate answers based on the units on the graph. Student values may vary slightly.

a) What is different about this graph?

The axes are labeled at certain intervals/values.

b) Given units on the graph, please rewrite your story with more details.

This could be a diver starting at a depth of approximately 15 feet for scuba diving. Then the diver could slowly go down another 10 feet over the course of 20 seconds. Next, the scuba diver could go another 15 feet in the course of 10 seconds. The scuba diver could then stop (for approximately 10 seconds) and look at the coral reef (or a water creature, etc.) and then rise back up to the surface.

c) The independent variable remains time, in seconds. What is the domain? Describe it in words and write it in interval notation.

The domain is 0 to 60 seconds, which would be written as [0, 60] in interval notation.

d) The dependent variable remains depth, in feet. What is the range? Describe it in words and write it in interval notation.

The range is 0 to 40 feet, which would be written as [0, 40] in interval notation.

e) Identify (approximately) the intercepts and describe their meaning in terms of the scenario.

$y$-intercept: (0, 15) At time 0 seconds, the depth is 15 feet. (The $y$- value may vary.)

$x$-intercept (60, 0) At 60 seconds, the depth is 0 feet.

f) Identify the intervals of increase, decrease, and constant and describe their meaning in terms of the scenario.

Increase: (0, 30) The scuba diver’s depth increased for the first 30 seconds.

Constant: (30, 40) The scuba diver’s depth remained constant for the next 10 seconds.

Decrease: (40, 60) The scuba diver’s depth decreased for the remaining 20 seconds.

Note: We use parentheses (open intervals) to show increasing, decreasing and constant intervals.

Suppose the piecewise function below represents the depth $D$ as a function of time $t$.

$$D(t)=0.5t+15 0\leq t\leq 20$$

 $1.5t-5 20<t<30$

 $40 30\leq t<40$

$$ -2t+120 40\leq t\leq 60$$

g) Evaluate $D(10)$, state the piece of the equation used to determine this value, and explain what it means in terms of the problem.

$D(10)=0.5(10)+15 = 20$

First piece

This means the depth is 20 feet at a time of 10 seconds.

h) Evaluate $D(35),$ state the piece of the equation used to determine this value, and explain what it means in terms of the problem.

$D(35)=40 $

Third piece

This means the depth is 40 feet at a time of 35 seconds.

i) Evaluate $D(40)$, state the piece of the equation used to determine this value, and describe why you used this piece.

$D(40)=-2(40)+120 = 40$

Fourth piece

For 40, you need to consider the final piece of the piecewise function. This is due to the 40 being included in the domain of the last piece (but not included in the third piece).

j) Suppose we want to solve $D(t)=35$. So we want to find the time(s) at which the depth is 35 feet.

i) A student says that a solution for $t$ is 40 because when $0.5t+15=35$ is solved for
$t$, then the value is 40. Is the student’s reasoning correct? Why or why not?

Although the student solved for $t$ correctly, the value does not lie in the domain of the piecewise equation piece. Therefore, the value of $t=40$ is incorrect.

ii) A student says that a solution for $t$ is $\frac{80}{3}≈$ 26.7 = because when $1.5t-5=35$ is solved for
$t$, then the value is $\frac{80}{3}$, and this number is between 20 and 30. Is the student’s reasoning correct? Why or why not?

This does give a correct solution for $t$ because the solution lies in the domain for the second piece of the piecewise equation. But it is not the only solution.

iii) Can any values $t\geq 30$ satisfy $D(t)=35$? Support your answer.

For $30\leq t<40$, $D(t)=40$ so there cannot be values of $t$ on this interval.

For $40\leq t<60,$ we could solve $-2t+120=35$ to determine if there is a value of $t$ that satisfies the function value of 35. Upon solving, we find $t=42.5$, which is allowable on this domain.

**Part Two:**

The graph shows the content spending, in billions of dollars, of Netflix worldwide from the year 2012 through 2024.



a) Describe the graph including key characteristics.

Answers may vary, but should include at least some of the following:

The graph represents a function. (Vertical Line Test)

The maximum amount of spending is 17.5 billion dollars, which occurred in 2021.

The minimum amount of spending is 1.75 billion dollars, which occurred in 2012.

The graph/content spending increased from 2012-2019, 2020-2021, and 2022-2023.

The graph/content spending decreased from 2019-2020 and again 2021-2022.

The graph/content spending was constant in 2023-2024.

The domain of the function is the years from 2012 to 2024.

The graph includes the year 2024, which most likely is a prediction since 2024 has not passed.

b) During which one-year period did the spending increase the most rapidly? Support your

answer with mathematics and describe it in terms of the scenario.

\*Note: 2012-2016 is not considered since it is the only non one-year period on the graph.

**2016-2017:** 8.91-6.88 = 2.03 **2017-2018:** 12.04-8.91 = 3.13

**2018-2019:** 14.6-12.04 = 2.56 **2020-2021:** 17.5-12.5 = 5

\*2020-2021 is the most rapid one-year period of increase. It is the portion of the graph with the greatest slope, not including 2012-2016.

**2022-2023:** 17-16.7 = 0.3 2019-2020: 12.5-14.6 = -2.1 \* Decreasing

2021-2022: 16.7-17.5 = -0.8 \*Decreasing 2023-2024: 17-17 = 0 \*Constant

c) Given $C(2020)=12.5$. Explain its meaning in terms of the scenario. What do you notice about this point on the graph?

In 2020, the content spending was 12.5 billion dollars. This point is a relative minimum. It is not the absolute minimum of the graph, but it is a low point compared to the points around it. It is possible that spending was lower this year due to the pandemic.

**Part Three:**

1) Fill in possible values in the table below based on the following information for a continuous function $f$.

a) The function has a $y$-intercept of $(0,-5)$. The function is increasing from $x=-4$ to $x=0$. The function is positive on the interval $[2,5]$. The function has an absolute maximum value at $x=2$.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$x$$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| $$f(x)$$ | *-100* | *-60* | *-42* | *-12* | -5 | 0 | *7* | *5* | *2* | *1* |

Answers may vary.

b) Based on the information from part (a), what can we conclude happens between $x=0$ and $x=2$?

Since the function is negative at $x=0$ and positive at $x=2$, then sometime between 0 and 2, there must be at least one $x$-intercept since $f$ is continuous.

c) Based on the information from part (a), can the function be increasing on the interval [2,5]? Explain your answer.

It cannot be increasing because the function has an absolute maximum at $x=5$.

d) Based on the information part (a), what can we say about what the function for $x>5$?

The only thing we can say is that the function never goes above the absolute maximum that occurs at $x=5$.

2) Suppose we have a continuous function $g$. Describe which key characteristic can be identified through the following description or given point.

a) Given $g(0)=9$.

This tells us the $y$-intercept of the function is $(0,9)$.

b) Given $g(6)=0$.

This tells us an $x$-intercept of the function occurs at $x=6$.

c) Given $g(-4)=5$ and $g(3)=-7$, what must occur between $x=-4$ and $x=3$? Can we say for sure that the function is decreasing on the interval $[-4,3]$?

Since the function is continuous, its graph must have an $x$-intercept on the interval $[-4,3],$ because it is positive at the left endpoint and negative at the right endpoint. The function also has a negative average rate of change on that interval, but it is not necessarily decreasing on the whole interval.