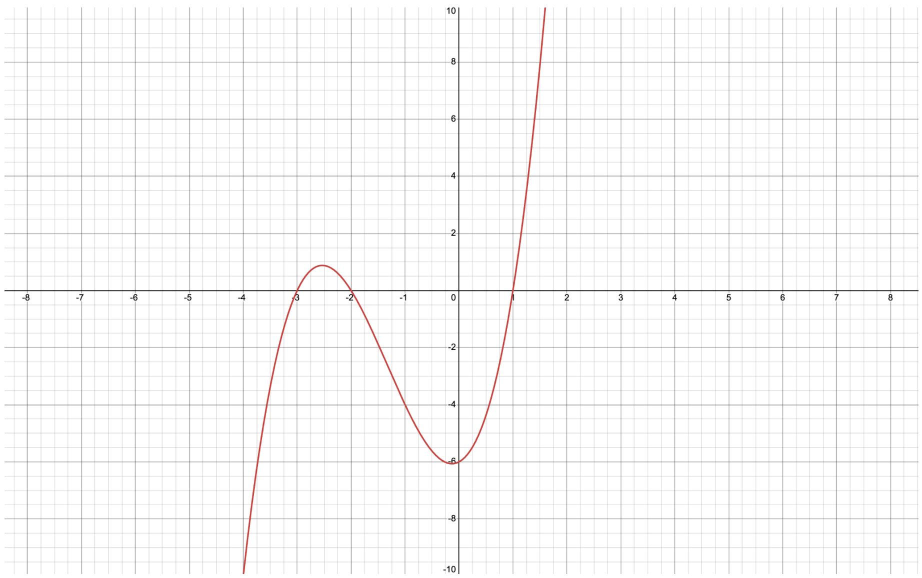
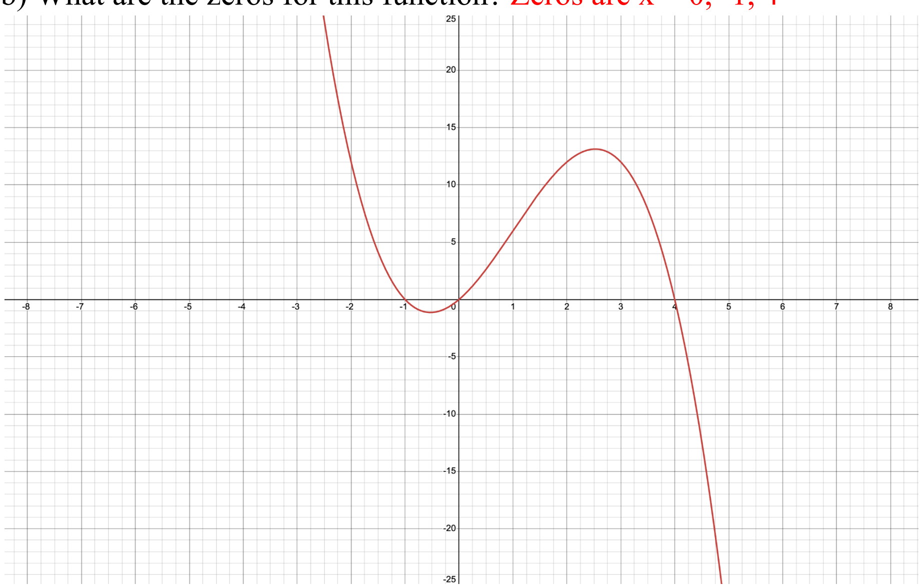
Linear Factors and Zeros of Polynomials
Name: ___________________ANSWER KEY_________________________ Date: ___________

Use Desmos to create the following graphs. For each function, sketch its graph and then state its zeros.

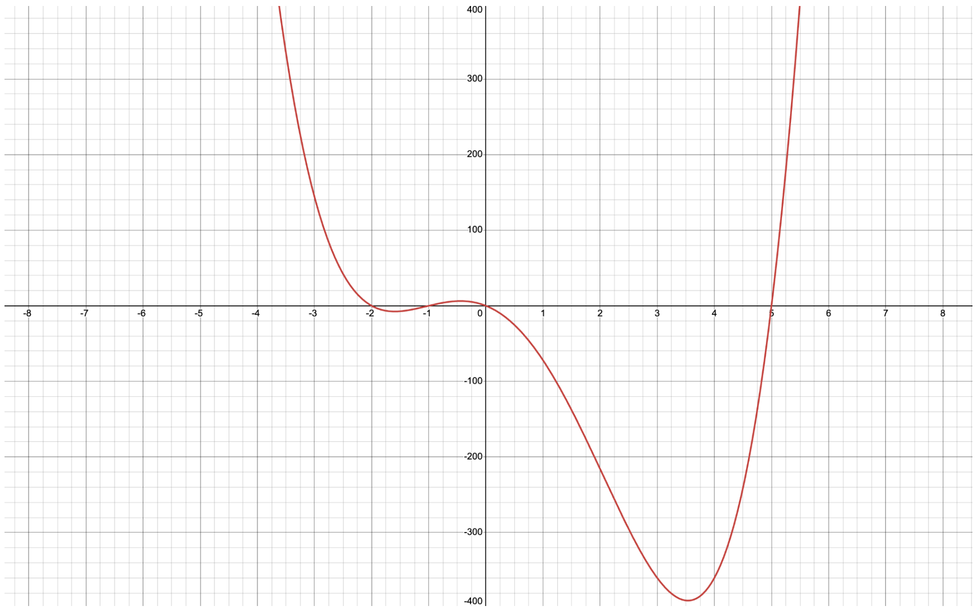
1. a) Graph $y = (x - 1)(x + 2)(x + 3)$ below. Note this function is the product of $y_1 = x - 1$, $y_2 = x + 2$, and $y_3 = x + 3$.
   b) What are the zeros for this function? Zeros are $x = 1, -2, -3$



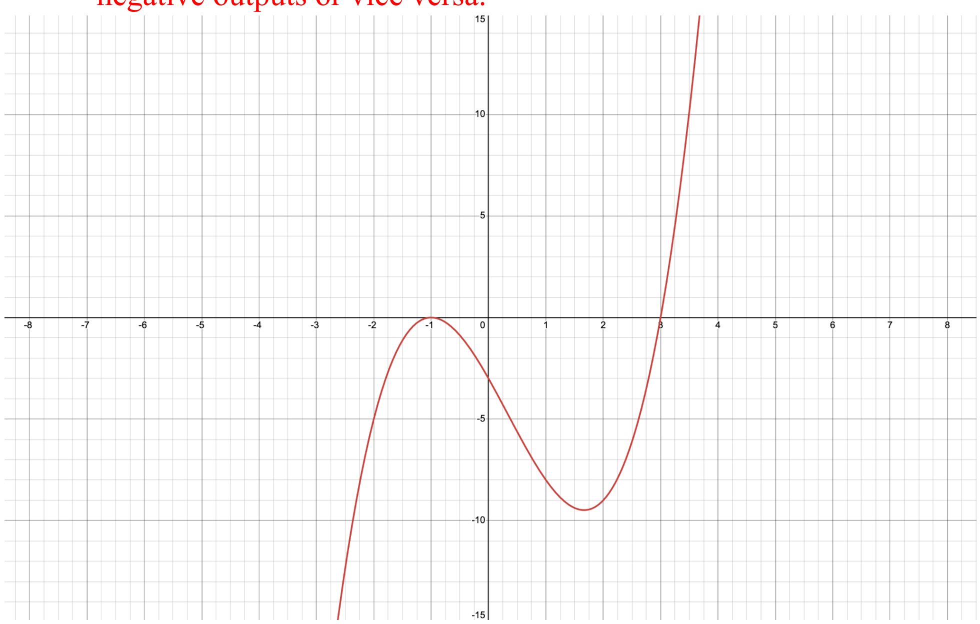
2. a) Graph $y = -x(x + 1)(x - 4)$ below. This function is the product of $y_1 = -x$, $y_2 = x + 1$, and $y_3 = x - 4$.
   b) What are the zeros for this function? Zeros are $x = 0, -1, 4$



3. Graph $y = 3x(x - 5)(x + 1)(x + 2)$. Note this function is the product of $y_1 = 3x$, $y_2 = x - 5$, $y_3 = x + 1$, and $y_3 = x + 2$.
   b) What are the zeros for this function? Zeros are $x = 0, 5, -1, -2$

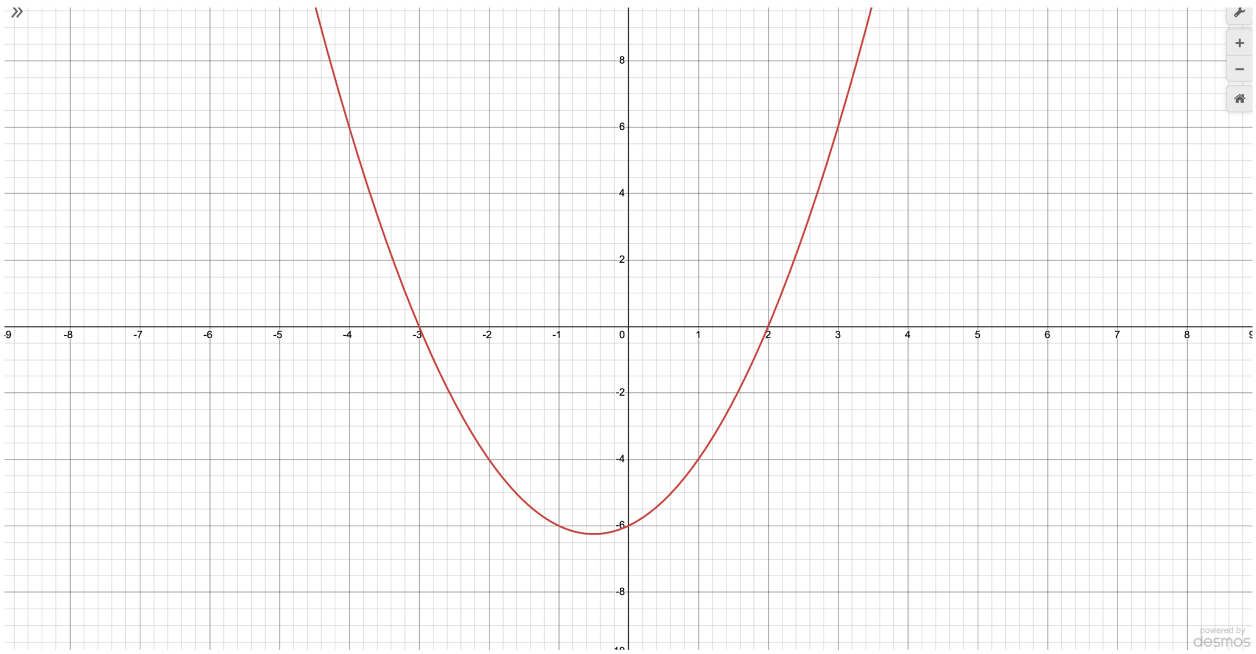


4. a) Graph $y = (x - 3)(x + 1)^2$. Note this function is the product of y_1, y_2, and y_3 where $y_1 = x - 3$ and $y_2 = y_3 = x + 1$. See the graph on the next page.
   b) What are the zeros for this function? Zeros are $x = 3, -1$
   c) Note that one of the linear factors, $(x + 1)$, is squared. How does its resulting zero differ from the zero associated with the linear factor $(x - 3)$? Answers will vary but are likely to mention that the graph's output values for this zero don't go from positive to negative outputs or vice versa.

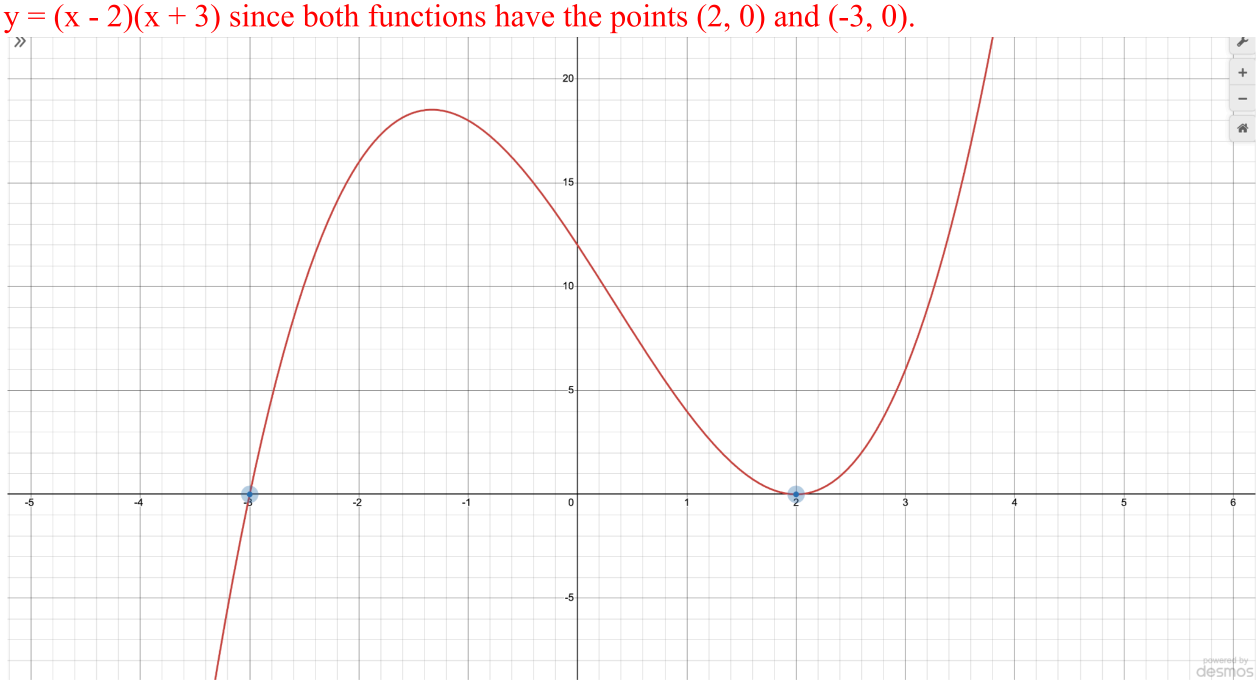


5. Compare the graph of y = (x - 2)(x+3) to each of the following graphs, one at a time. State how the graph each is similar and how each is different.
   a. $a(x) = (x - 2)^2(x + 3)$
   b. $b(x) = (x - 2)(x + 3)^2$
   c. $c(x) = (x - 2)^2(x + 3)^2$
   d. $d(x) = (x - 2)^2(x + 3)^3$

The graph of y = (x - 2)(x+3) is below. It has zeros at x = 2 and x = -3.

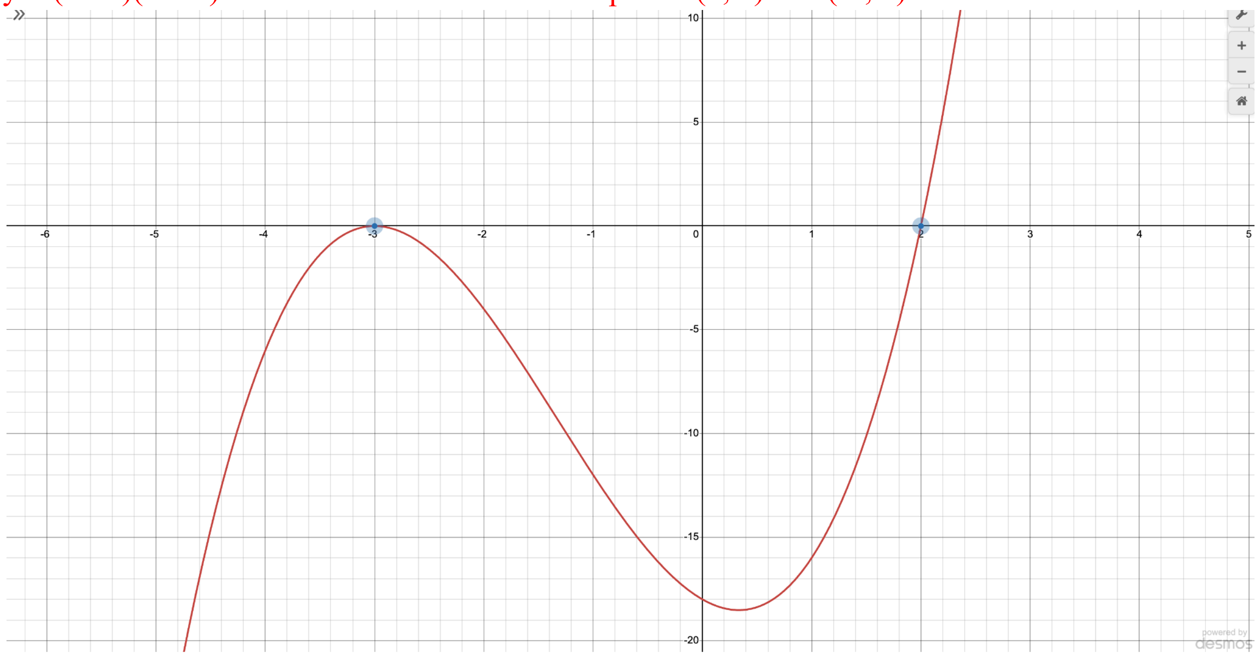


a. Sample answer, there are many.
   The graph of $a(x) = (x - 2)^2(x + 3)$, as shown below, has the same zeros as the function y = (x - 2)(x + 3) since both functions have the points (2, 0) and (-3, 0).



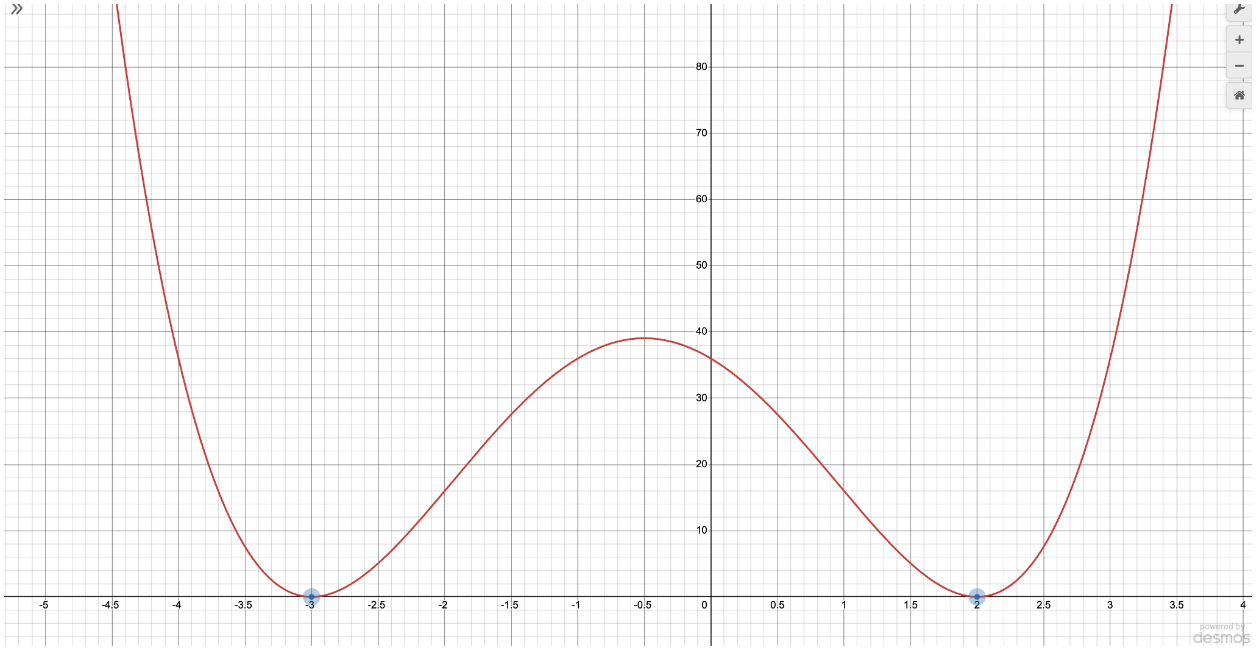
However, the linear factor (x - 2) has a multiplicity of 2. So, a(x) only touches at the point (2,0) and the outputs in the function go from being positive to zero then back to positive; so, the graph "rebounds" at this zero. Some students may also comment that close to x = 2 the graph of a(x) looks or acts like a parabola.

b. Sample answer, there are many.
The graph of $b(x) = (x - 2)^2(x + 3)^2$, as shown below, has the same zeros as the function y = (x - 2)(x + 3) since both functions have the points (2, 0) and (-3, 0).



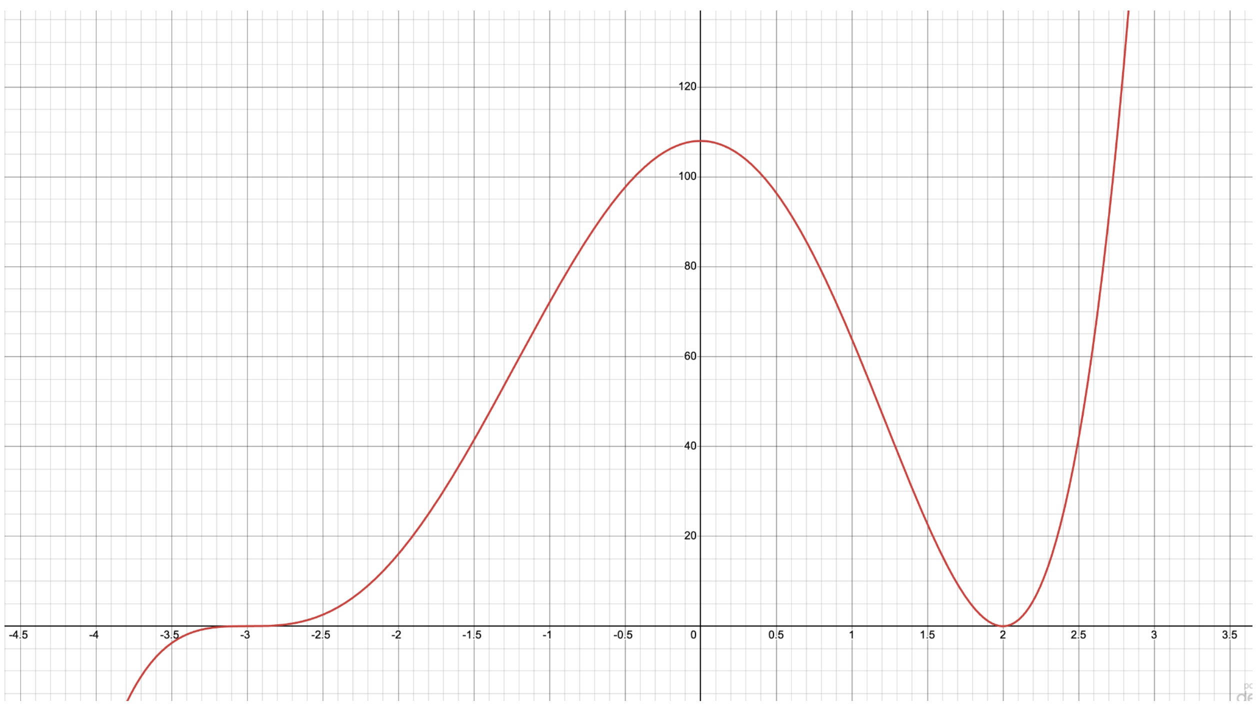
However, both of the linear factors (x + 3) and (x - 2) each has a multiplicity of 2. So, c(x) only touches at the point (-3,0) and the outputs in the function go from being negative to zero then back to negative; so, the graph "rebounds" at this zero. Some students may also comment that close to x = -3 the graph of b(x) looks or acts like a parabola.

c. Sample answer, there are many.
The graph of $c(x) = (x - 2)^2(x + 3)^2$, as shown below, has the same zeros as the function y = (x - 2)(x + 3) since both functions have the points (2, 0) and (-3, 0).



However, the linear factor (x + 3) has a multiplicity of 2. So, b(x) only touches at the point (-3,0) and the outputs in the function go from being positive to zero then back to positive while b(x) only touches at (2, 0 with the outputs going from positive to zero then back to positive. Some students may also comment that close to both x = -3 and then again close to x = 2, the graph of c(x) looks like a parabola.

d. Sample answer, there are many.
The graph of $d(x) = (x - 2)^2(x + 3)^3$, as shown below, has the same zeros as the function $y = (x - 2)^2(x + 3)$ since both functions have the points (2, 0) and (-3, 0).



However, the linear factor (x + 3) has a multiplicity of 3, and the linear factor (x - 2) has a multiplicity of 2. So, d(x) looks and acts like a parabola right around x = 2, but it looks and acts more like a cubic function right around the input value of x = -3.