LESSON TITLE: The Rule of 70

OVERVIEW: This lesson explores the Rule of 70, a financial mathematics concept that estimates the time it takes for an investment to double when interest is compounded continuously. Students will use both algebraic methods and an interactive Desmos activity to understand and apply the rule, enhancing their understanding of logarithms, exponential growth, and financial literacy.

PREREQUESITE IDEAS AND SKILLS:

- Familiarity with the concept of continuously compounded interest
- Understanding of logarithms and their use in solving equations
- Basic calculator and computer skills for using Desmos

MATERIALS NEEDED TO CARRY OUT THE LESSON

- Activity worksheets
- Computers or tablets with internet access for Desmos
- Calculators (optional)

CONCEPTS TO BE LEARNED/APPLIED

- Students will understand the Rule of 70 is "if an amount of money grows at an annual interest rate of $r\%$, compounded continuously, it will take roughly $\frac{70}{r}$ years to double" because solutions to the doubling time equation involve $\ln(2)$, which is about 0.70.
- Students will understand that the doubling time is independent of the starting amount because the doubling time equation $2P = Pe^{\left(\frac{r}{100}\right)t}$ simplifies to $2 = e^{\left(\frac{r}{100}\right)t}$. In other words, the initial value disappears from the equation.
- Students will apply the Rule of 70 to various financial scenarios, making estimates about the growth of investments over time, thus enhancing their financial literacy and understanding of exponential growth.
- Students will explore variations of the Rule of 70, including comparison with the Rule of 72, and extension to concepts like tripling time.

STRUCTURE OF THE CONCEPTS TO BE LEARNED

- The Rule of 70 states: "If an amount of money grows at an annual interest rate of $r\%$, compounded continuously, it will take roughly $\frac{70}{r}$ years to double." As an example, if $r = 5\%$, it will take about 14 years to double (i.e., $\frac{70}{5} = 14$). The mathematical reason this rule works boils down to the fact that $\ln(2) \approx 0.70$, and algebraic solutions to the doubling time equation $2P = Pe^{\left(\frac{r}{100}\right)t}$ have the form $\frac{\ln(2)}{\frac{r}{100}} = \frac{100\ln(2)}{r} \approx \frac{70}{r}$.
- The Rule of 70 depends on the interest rate r, but not on the initial amount of money, because the doubling time equation $2P = Pe^{\left(\frac{r}{100}\right)t}$ can be simplified (by dividing both sides by P) to $2 = e^{\left(\frac{r}{100}\right)t}$. In other words, the initial value disappears from the equation.

INSTRUCTIONAL PLAN

The lesson begins with a recollection of the formula for continuously compounded interest: $A(t) = Pe^{rt}$, where P is the principal amount, r is the annual interest rate expressed as a decimal, and t is time in years. The activity then presents the following scenario: "Suppose $10,000 is deposited in a bank account that earns 5% annual interest, compounded continuously. How long will it take for the money to double?"

Students engage in a Desmos activity, which is accessible through a provided link, where they interact with a dynamic graph representing the growth of an investment. The activity is designed with sliders for the initial amount P and the interest rate r, allowing students to observe the changes in the investment's growth over time. Two specific ordered pairs are highlighted in the activity, one in red and one in black, representing the initial investment and its doubled amount, respectively. As students engage with the Desmos sliders, adjusting the initial amount and interest rate, they will observe the graphical changes and the corresponding doubling times. This interactive component is important for facilitating an understanding that the doubling time is independent of the initial investment amount but varies inversely with the interest rate.

Students then fill out their worksheets, noting the doubling times for predetermined interest rates, thereby leading them to discover the pattern that lies at the foundation of the Rule of 70.

The lesson then shifts to an algebraic approach, where students are tasked with solving the initial question algebraically using logarithms (i.e., solving the equation $20{,}000 = 10{,}000e^{0.05t}$ for t). Students are then asked how the solution would change for different initial amounts (i.e., it won't change) and for different interest rates. Students are thus led to discover the Rule of 70 algebraically, showing that $t \approx \frac{70}{r}$ when r is expressed as a percentage. This rule is derived from the fact that $100 \cdot \ln(2) \approx 70$, which greatly simplifies the calculation of doubling time (without a calculator). Students apply this reasoning to various interest rates, further solidifying their understanding of the rule.

To assess understanding, students apply the Rule of 70 to different scenarios, such as predicting the growth of a $30,000 investment at a 10% interest rate or determining the value of a $100,000 investment at a 7% interest rate after 24 years. They also compare the Rule of 70 to the Rule of 72, discussing why one might be used over the other, and explore the concept of tripling time by considering the natural logarithm of 3.

The goal of the lesson is not merely for students to understand what the Rule of 70 is and how to apply it in practical situations, but to understand the underlying mathematics of why the rule works.

MIP COMPONENTS OF INQUIRY

This section outlines how our activity will meet the Mathematical Inquiry Project (MIP) criteria for active learning, meaningful applications, and academic success skills.

Active Learning: This activity will engage students in active learning as they discover the Rule of 70. Students use Desmos in a way that allows them to make changes to various inputs and to see the results immediately. For example, students move a slider to change the initial amount, and they engage in the mental activity of evaluating whether or not the Δx between the points of intersection changes. Noticing that the Δx is fixed, they conclude that changing the initial amount does not change the doubling time. They will then create a table of values using the Desmos activity and discover a pattern in the table that relates the interest rate and the doubling time in a way that involves the number 70. In this way, students actively discover the Rule of 70 rather than simply being given the rule. Students also engage in the process of solving the doubling time equation algebraically (using logarithms) and evaluating that solutions have the form $\frac{100\ln(2)}{r}$, which is approximately $\frac{70}{r}$ because $\ln(2) \approx 0.70$.

Meaningful Applications: This activity will emphasize meaningful applications involving the Rule of 70 and financial mathematics. Students will utilize their prior knowledge of solving exponential equations using logarithms to discover the Rule of 70 algebraically. Students will also see graphically (using Desmos) how the rule works. They will also make generalizations by, for example, coming up with a "Rule of 110" for tripling-time. Students will understand not only how to properly use the Rule of 70 in real-world situations, but they will understand why the rule works.

Academic Success Skills: This activity will allow students to enhance their identities as learners by recognizing their capability to make important mathematical discoveries themselves rather than simply being given formulas. Moreover, students will be empowered to make quick and easy calculations in their head for real-world complex financial mathematics, thereby enhancing their financial literacy. Since students are making discoveries themselves, they may build more confidence in their mathematical abilities and view math as useful in their lives beyond the classroom.