Properties of Exponents Discovery

Part 1

1. Circle two ways to write this expression in a shorter notation. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

   a) $3 \cdot 5$ b) 3^5 c) 15

   d) 243 e) 5^3 f) $5 \cdot 3$

2. Using the same idea, write this expression in a shorter notation.

   $x \cdot x \cdot x \cdot x \cdot x \cdot x$

3. Write the expression as a multiplication problem with two bases.

   a) $(y \cdot y) \cdot (y \cdot y \cdot y \cdot y \cdot y \cdot y)$ b) $(7 \cdot 7) \cdot (7 \cdot 7 \cdot 7)$

   c) Now, write the expression into a shorter notation with only one base.

   d) Notice the relationship between parts (a) & (b). Write a rule about exponents when multiplying with the same base.

4. Simplify the following expressions using the rule you discovered in questions 1 - 3.

a) $(x^3)(x)$ b) $x^3y^4x^2y^5$ c) $4^3 \bullet 4^2$

d) $2 \bullet x^7 \bullet 2^3 \bullet x^5$ e) $(x + 2)^2 (x + 2)^4$

5. What do you think would happen to the rule if one of the exponents was negative?

6. Simplify the following expressions.

a) $x^7 \bullet x^{-4}$ b) $y^{-4}(y^5)$ c) $5^4 \bullet 5^{-2}$ d) $(x + 2)^{-2}(x + 2)^5$

7. You have two square areas with sides measuring $2x^3$ for the first square and $7y^4$ for the second square Using the formula for the area of a square $A = s^2$, find the area of each square by applying expansion and the exponential rules you have discovered.

Part 2

1. Use your knowledge of expanding an exponential expression to write each expression in expanded form. (Leave them as fractions)

   a) $\frac{x^7}{x^5}$ b) $\frac{s^5}{s^2}$ c) $\frac{3^7}{3^2}$ d) $\frac{h^7w^4}{h^3w}$

2. Now simplify each expanded expression into the shortest possible expression.

   a) b) c) d)

3. Notice the relationship between parts (1) & (2). Write a rule about exponents when dividing with the same base.

4. Simplify the following expressions using the rule you discovered in questions 1 - 3.

   a) $\frac{x^3y^5w^2}{x^2y^3w}$ b) $\frac{3^7x^4y^8}{3^3x^3y^5}$ c) $\frac{(2x+3)^7}{(2x+3)^3}$ d) $\frac{(3y+4)^9(y-4)^2}{(3y+4)^4}$

5. Comparing two values exponentially with the same base can be done using the rule you have just discovered. This is known as the quotient rule. So you can determine how many times greater one quantity, value, or distance is than another if they have the same base by dividing the two values and applying the quotient property. Using the rule you have discovered in Part 2 determine how many times greater the first value is compared to the second in each example.

   a) an earthquake in South America has an intensity of $10^{6.5}$ and another in California has an intensity of $10^{4.1}$. How many times greater is the South American earthquake?

   b) On a specific day the Mercury is 3.84×10^{7} miles from the sun. On that same day Saturn is 8.86×10^{8} miles from the sun. How many times further is Saturn than the Mercury from the sun?

Part 3

1. Expand the expression completely into a multiplication problem with no exponents.

   a) $(2xy)^2$

   b) $(3w^3y^2)^3$

   c) $(5^2m^2p^3)^2$

2. Simplify the expanded expressions in question 1 so each coefficient or variable base has only one exponent. For example, $4{\cdot}4{\cdot}x{\cdot}x{\cdot}x{\cdot}y{\cdot}y\cdot y = 4^2x^3y^3$

   a)

   b)

   c)

What do you notice happened to the exponents for each coefficient or variable base from question 1 to question 2?

3. Using the combined methods from questions 1 & 2, write each of the following so each coefficient or variable base has only one exponent.

a) $(2x^2y^3)^2$

b) $(4w^4y^5)^3$

c) $(3^2m^5p^2)^4$

Notice the relationship between parts (1) & (2). Write a rule about exponents when a base has an exponent raised to another exponent.

4. Give examples of different ways can you write x^{12} in the form of $(x^{\square})^{\square}$ (think outside the box and not use just positive exponents)

5. You have discovered the rule for exponents raised to other exponents which we call powers raised to powers? Use that rule to simplify the following expressions.

a) $(2^2xy^3)^4$ b) $(5v^4w^3)^2$ c) $[(s-7)^3]^4$

Part 4

Following discussion in groups and with the instructor, use the rules discovered to find the errors involving exponents and make the corrections.

1. $(x^2)^3 = x^5$ 1.

2. $x^3 \bullet x^4 = x^{12}$ 2.

3. $(2x)^3 = 2x^3$ 3.

4. $(3y)^4 = 12y^4$

5. $\frac{w^6}{w^2} = w^3$

Part 5

Find the area of a rectangle with length of $(2zz^2)$ and a width of $(5xxy^3y^2)$

(Area = lw)

The area of a rectangle is $(27m^{12}p^6)$. If the length of the rectangle is $(3m^8p^3)$, find the width.