LESSON TITLE: Exploring the Connection Between Secant Lines and the Tangent Line at a Given Point

OVERVIEW: This lesson focuses on the connection between secant lines and the tangent line at a given point. Students will begin with a given quadratic function and a point on the graph. They will explore the slopes of various lines secant to the given graph by hand, as well as via a Desmos activity. Through various investigations of the secant lines and their slopes, students are encouraged to predict the slope of the graph at the given point. Afterward, they compute the actual slope of the graph at that point using the limit of a difference quotient. Students then have the opportunity to change the given point or the given function, make new predictions regarding the slope of the graph at a point, and check their work in Desmos.

PREREQUISITE IDEAS AND SKILLS:

- Evaluate a function at a given value
- Compute a limit
- Understand that the average rate of change is a relationship between two varying quantities in a dynamic situation
- Ability to follow the steps in a packet of directions

MATERIALS NEEDED TO CARRY OUT THE LESSON:

- Calculator
- Student access to internet to utilize Desmos
- In-Class Activity Worksheet
- Link to Desmos Activity (embedded in Worksheet)

CONCEPTS TO BE LEARNED/APPLIED:

- Students will understand that for a differentiable function, zooming in on the function at a particular point will cause the function to appear more linear near that point, and the slope of this line is the slope of the graph at that point.
- Students will understand that the average rate of change corresponds to the slope of a secant line.
- Students will understand that the slopes of secant lines through a given point on a graph will approach the slope of the graph at that point.
- Students will understand that the secant lines through a given point will approach the tangent line of the graph at that point.
- Students will understand that the instantaneous rate of change at a point corresponds to the slope of the tangent line at that point.
- Students will understand that the slope of a tangent line can be found using the limit of a difference quotient.

INSTRUCTIONAL PLAN:

The instructor will provide the activity worksheet to the students. If students also have access to the worksheet online, they can follow the given link to the Desmos activity. Otherwise, access to the link to the Desmos activity needs to be provided. Students will begin viewing a given graph in Desmos and predicting the slope of the graph at a given point by zooming in on that point. They will then complete a table of *x*- and *y*-values of additional points that are "close" to the given point, as well as the slope of the secant lines created between those points and the given point. Using the table, students will predict the slope of the graph at the given point and compare it to their previous prediction from Desmos. Students will then compute the slope of the graph at the given point by taking the appropriate limit of a difference quotient. Again, they will compare this answer to their predictions. They will then find the equation of the tangent line to the graph at the given point. Students complete the activity by computing the tangent line to a function and point of their choosing and check their results in Desmos.

MIP COMPONENTS OF INQUIRY:

Active Learning: In this activity, students will consider the graph of a function, estimate the slope of the graph at a given point, compute the slope of select secant lines, develop the relationship between the slopes of the secant lines and the slope of the tangent line at the given point, and ultimately, compute the equation of a tangent line. Students will engage in active learning as they recall that the average rate of change is the quotient of the change of *y*-values and the change in *x*-values. Students will be using various representations including a table of *x*- and *y*-values, as well as a graph provided in Desmos. In Desmos, they will use the "zoom" feature by "zooming in" until the output quantity appears to vary at a constant rate with respect to the input quantity. Afterwards, they are asked to notice the multiplicative comparison of these corresponding changes. As the students proceed through the activity, they will generalize the quantities used in the difference quotient by writing expressions to represent various graphical quantities. At the end of the activity, students will create and explore their own graphs in Desmos and solidify the connections mentioned above.

Meaningful Applications: This activity enhances meaningful applications as students will have the opportunity to explore and identify relationships between the slopes of lines secant to a graph through a certain point and the slope of the graph at that point. As they work through the activity, the connection between the slope of a secant line and the average rate of change will be made, as well as the connection between the slope of a tangent line and the instantaneous rate of change. This will be accomplished as students utilize various representations to make and justify claims regarding the slope of a graph at a point and hence the slope of the tangent line at that point. This activity is also meaningful as the students will use their knowledge of limits to make their claims and ultimately, to compute a derivative.

Academic Success Skills: Students will be able to apply this activity to their choice of a function and a point. As they work towards finding the equation of the tangent line, they will be able to check their work in the Desmos activity. By allowing students the opportunity to use Desmos, they will be able to visualize and check their work as they progress towards finding the equation of the tangent line. This reinforcement will provide students feedback and an opportunity to reflect on their work throughout the activity. Students will also be able to explore a larger variety of graphs, secant lines, and tangent lines through the Desmos activity as they will not have to compute everything by hand which in turn, could reduce mathematical anxiety.