**Inverse Trigonometric Functions and Graphs**

*Description*: This activity aims to extend students’ understanding of inverse functions and their graphs to the class of trigonometric functions (specifically: sine, cosine, and tangent). We address the development of restricted domains for the inverse functions, and encourage exploration of that idea.

**Possible Learning Objectives**

* MA 203.8 and MA 204.4: Identify properties and characteristics of trigonometric functions,
* specifically one-to-one and inverse
* MA 203.4: Interpret functions and convert between their representations
* MA 204.2: Construct and interpret graphs of trig functions and their transformations

**Activity Objectives**

The students working on this activity will work towards understanding:

* How to construct inverse functions from given functions.
* How to choose input intervals that produce inverse functions.
* Why certain inverse function input values will produce “unexpected” output values.

**Prior Knowledge Assumed**

Before completing this activity, students should have encountered or mastered the following:

* General definition of one-to-one
* Domain and range of trigonometric functions
* Non-transformed graphs of sine, cosine, and tangent

It would also be ideal, but not necessary if the following have been discussed:

* The graphs of secant, cosecant, and cotangent

**Learning Outcomes**

* *Measurable Outcomes*
  + Students will determine necessary conditions for the creation of an inverse function, including domain restriction and the definition of one-to-one.
  + Students will also identify intervals in which a function is not one-to-one.
  + Students will use the graph of a function to create the graph of the inverse function.
  + Students will attend to the above outcomes in the context of trigonometric functions.
* *Extensions*
  + Students can use the process described to determine a restricted domain for cotangent, secant, and cosecant so that inverse functions can be created.
  + Students will recognize reasonable solutions to inverse trig expressions, differentiating between input values (“distances”) and output values (“rotations”).

**Elements of Mathematical Inquiry**

* *Active Learning*

In the process of developing solutions for inverse trig functions, students often memorize and mimic specific values rather than understand them or where they derive. This activity forces students to determine what measure(s) must be taken in order to find an inverse function, and specifically how this relates to trigonometric functions. They are explicitly asked to evaluate their process along the way, leaving both selecting and performing tasks which satisfy the instructions to the student, ideally resulting in a more robust understanding of inverse trig solutions.

* *Meaningful Application*

Students are asked to explicitly draw connections between an original function and its inverse, first for a standard parabola, then with trig functions. This helps bridge understanding between concept ideas of the two function families. Additionally, because students are asked to work with more than one restricted domain, they must justify to themselves and others (groupmates, classmates, their instructor, etc.) why a certain solution exists in that restricted domain.

* *Academic Success Skills*

By understanding the reasoning behind traditional restricted domains, students’ anxiety about working with non-traditional restricted domains could be lessened. Additionally, this activity supports problem solving and critical thinking by forcing students to justify the validity of their answers. This helps develop classroom communities by establishing interdependence and trust. The activity is built to encourage productive struggle, persistence, and perseverance by asking open-ended questions meant to challenge students’ assumptions.

**Recommended Technologies**

Students should have a scientific calculator that can compute inverse trig values.