Properties of Exponents Discovery

Part 1

1. Circle two ways to write this expression in a shorter notation. $3 \bullet 3 \bullet 3 \bullet 3 \bullet 3$

a) $3 \bullet 5$ b) 3^5 c) 15

d) 243 e) 5^3 f) $5 \bullet 3$

b) 3^5 d) 243 both are equal to the expanded expression

This is a good chance from the start to emphasize to students the possibility of multiple equivalent expressions.

2. Using the same idea, write this expression in a shorter notation.

$x \bullet x \bullet x \bullet x \bullet x \bullet x$

x^6

Students should understand that all terms are the same base so they can be written as one base with the exponent of 6.

3. Write the expression as a multiplication problem with two bases.

a) $(y \bullet y) \bullet (y \bullet y \bullet y \bullet y \bullet y \bullet y)$ b) $(7 \bullet 7) \bullet (7 \bullet 7 \bullet 7)$

a) $y^2 \bullet y^6$ b) $7^2 \bullet 7^3$

Students should only be combining the terms within the parenthesis.

c) Now, write the expression into a shorter notation with only one base.

a) y^8 b) 7^5

Students should be aware that the exponent represents the total times the base is multiplied together. They should also be drawing conclusions about multiplying 2 separate but identical bases each with exponents to reach a product with one base.

d) Notice the relationship between parts (a) & (b). Write a rule about exponents when multiplying with the same base.

Student rules should include some explanation of adding exponents when multiplying powers with the same base.

Instructor Note: In promoting student's academic Success skills, have them discuss specific terminology they have used in their definitions to develop a mathematical explanation creating a clear and concise exponent rule. For Active Learning ask students to apply their rule to each previous and future example to ensure it actually does apply to all examples correctly.

4. Simplify the following expressions using the rule you discovered in questions 1 - 3.

a) $(x^3)(x)$

x^4

b) $x^3y^4x^2y^5$

x^5y^9

c) $4^3 \bullet 4^2$

4^5

d) $2 \bullet x^7 \bullet 2^3 \bullet x^5$

2^4x^{12}

e) $(x + 2)^2 (x + 2)^4$

$(x + 2)^6$

For parts (c) and (d) students should keep the numerical values with exponents. So even though $4^5 = 1024$ the concept here is not evaluating exponents but rather discovering and applying the exponential properties. Be sure to help students realize that the binomial base is treated just like a numerical or variable base. The goal here is not to expand the binomial but just apply the exponential rule.

5. What do you think would happen to the rule if one of the exponents was negative?

The idea here is for students to realize that the same property applies and they are still multiplying with the same base so they would still add the 2 exponents regardless of their sign.

6. Simplify the following expressions.

a) $x^{7} \bullet x^{-4}$ b) $y^{-4}(y^{5})$ c) $5^{4} \bullet 5^{-2}$ d) $(x + 2)^{-2}(x + 2)^{5}$

a) x^{3} b) y c) 5^{2} *d)* $(x + 2)^{3}$

This is a good time to emphasize to students that x or y as a single variable is the same as x^{1} or y^{1}, so either form is correct and if it helps students' understanding to include the exponent of 1 then they can choose to do that . However, point out that the accepted mathematical notation is just the variable by itself and the exponent of 1 is understood. This will show up in future problems as well later in this activity.

7. You have two square areas with sides measuring $2x^{3}$ for the first square and $7y^{4}$ for the second square Using the formula for the area of a square $A = s^{2}$, find the area of each square by applying expansion and the exponential rules you have discovered.

Square 1 $A = s^{2} = (2x^{3})^{2} = 2x^{3} \bullet 2x^{3} = 2 \bullet 2 \bullet x^{3} \bullet x^{3} = 2^{2}x^{6} = 4x^{6}$

Square 2 $A = s^{2} = (7y^{4})^{2} = 7y^{4} \bullet 7y^{4} = 7 \bullet 7 \bullet y^{4} \bullet y^{4} = 7^{2}y^{8} = 49y^{8}$

Students should be using the same ideas that they have previously used in the discovery portion or Part 1. Even though the power to a power property could also be applied here, this problem should stress adding exponents with the same base which is why the expansion is also mentioned. Ensure that students understand that the exponent of 2 is applied to both the numerical and variable bases.

Instructor Note: Applying the rules that have worked on in the Active Participation of Part 1 should help to build their confidence in the Academic Success Skills and mathematical application of exponents. Instructors should be sure to stress the mathematical application of exponents to help students realize the meaningful application surpasses just simplifying an exponential expression and can be applied in future problems involving exponents and other mathematical processes.

Part 2

1. Use your knowledge of expanding an exponential expression to write each expression in expanded form. (Leave them as fractions)

a) $\frac{x^7}{x^5}$ b) $\frac{s^5}{s^2}$ c) $\frac{3^7}{3^2}$ d) $\frac{h^7w^4}{h^3w}$

a) $\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$ *b)* $\frac{s \cdot s \cdot s \cdot s \cdot s}{s \cdot s}$ *c)* $\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$ *d)* $\frac{h \cdot h \cdot h \cdot h \cdot h \cdot h \cdot h \cdot w \cdot w \cdot w \cdot w}{h \cdot h \cdot h \cdot w}$

2. Now simplify each expanded expression into the shortest possible expression.

a) b) c) d)

a) x^2 b) s^3 c) 3^5 d) h^4w^3

Once again example c) should be written with exponents rather than as 243 and the w in the denominator of example d) is understood as w^1

3. Notice the relationship between parts (1) & (2). Write a rule about exponents when dividing with the same base.

Student's rule should include an explanation of subtracting the exponents of each base when dividing powers with the same base.

Instructor Note: In promoting student's academic Success skills, have them discuss specific terminology they have used in their definitions to develop a mathematical explanation creating a clear and concise exponent rule. For Active Learning ask students to apply their rule to each previous and future example to ensure it actually does apply to all examples correctly.

4. Simplify the following expressions using the rule you discovered in questions 1 - 3.

a) $\frac{x^3y^5w^2}{x^2y^3w}$ b) $\frac{3^7x^4y^8}{3^3x^3y^5}$ c) $\frac{(2x+3)^7}{(2x+3)^3}$ d) $\frac{(3y+4)^9(y-4)^2}{(3y+4)^4}$

a) xy^2w b) 3^4xy^3 c) $(2x+3)^4$ d) $(3y+4)^5(y-4)^2$

Ensure that students are applying their rule to any powers with the same base both variable and numerical bases. By this point students who are still writing x^1 or w^1 should be reminded that the accepted mathematical notation is to leave off the 1 exponent.

5. Comparing two values exponentially with the same base can be done using the rule you have just discovered. This is known as the quotient rule. So you can determine how many times greater one quantity, value, or distance is than another if they have the same base by dividing the two values and applying the quotient property. Using the rule you have discovered in Part 2 determine how many times greater the first value is compared to the second in each example.

a) an earthquake in South America has an intensity of $10^{6.5}$ and another in California has an intensity of $10^{4.1}$. How many times greater is the South American earthquake?

$$\frac{10^{6.5}}{10^{4.1}} = 10^{6.5-4.1} = 10^{2.4} = 251.1886 \approx 251.2 \textit{ times greater}$$

b) On a specific day the Mercury is 3.84 $\times 10^7$ miles from the sun. On that same day Saturn is 8.86 x 10^8 miles from the sun. How many times further is Saturn than the Mercury from the sun?

$$\frac{8.86 \, x \, 10^{8}}{3.84 \, x \, 10^{7}} = \frac{8.86}{3.84} x \, 10^{8-7} \approx 2.31 \, x \, 10^{1} \approx 23.1 \text{ times closer to the sun}$$

Ensure that students relate the property of subtracting the exponents when the bases are the same. Instructors may need to provide some assistance in part b.

Going further with these ideas, having a class or group discussion about other areas that may be applicable will allow students to expand on the meaning applications while building their Academic Success skills by engaging in academic discussion and thoughtful reasoning.

Part 3

1. Expand the expression completely into a multiplication problem with no exponents.

a) $(2xy)^2$

$2 \cdot 2 \cdot x \cdot x \cdot y \cdot y$

(notice that in this example each base is starting with an understood exponent of 1)

b) $(3w^3y^2)^3$

$3 \cdot 3 \cdot 3 \cdot w \cdot w \cdot w \cdot w \cdot w \cdot w \cdot w \cdot w \cdot w \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

c) $(5^2m^2p^3)^2$

$5 \cdot 5 \cdot 5 \cdot 5 \cdot m \cdot m \cdot m \cdot m \cdot p \cdot p \cdot p \cdot p \cdot p \cdot p$

2. Simplify the expanded expressions in question 1 so each coefficient or variable base has only one exponent. For example, $4 \cdot 4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y = 4^2x^3y^3$

a) $2^2x^2y^2$

b) $3^3w^9y^6$

c) $5^4m^4p^6$

What do you notice happened to the exponents for each coefficient or variable base from question 1 to question 2?

Students should be seeing that the exponents are being multiplied together.

3. Using the combined methods from questions 1 & 2, write each of the following so each coefficient or variable base has only one exponent.

a) $(2x^2y^3)^2$ $\quad 2^2x^4y^6$ or $4x^4y^6$

b) $(4w^4y^5)^3$ $\quad 4^3w^{12}y^{15}$ or $64w^{12}y^{15}$

c) $(3^2m^5p^2)^4$ $\quad 3^6x^{20}y^8$ or $729x^{20}y^8$

Notice the relationship between parts (1) & (2). Write a rule about exponents when a base has an exponent raised to another exponent.

Student's rule should include an explanation of raising an exponent to another exponent and rewriting each base with only 1 exponent. Coefficients can be simplified depending on the instructors choosing.

4. Give examples of different ways can you write x^{12} in the form of $(x^{\square})^{\square}$

(think outside the box and not use just positive exponents)

Possible solutions – Instructors should encourage students to find each combination of exponent answers. $(x^1)^{12}$ $(x^{12})^1$ $(x^2)^6$ $(x^3)^4$ $(x^6)^2$ $(x^4)^3$ $(x^{-4})^{-3}$ $(x^{-2})^{-6}$

Other combinations of negative exponents could work for these solutions.

5. You have discovered the rule for exponents raised to other exponents which we call powers raised to powers? Use that rule to simplify the following expressions.

a) $(2^2xy^3)^4$ b) $(5v^4w^3)^2$ c) $[(s-7)^3]^4$

$2^8x^4y^{12}$ $5^2v^8w^6$ $(s-7)^{12}$

Ensure that students are applying the exponents to both numerical and variable bases. Once again the numerical values have been left in exponential form. While 256 for part a) or 25 for part b) is not wrong, the focus here is applying the exponential rules.

Be sure to help students realize that the binomial base is treated just like a numerical or variable base. The goal here is not to expand the binomial but just apply the exponential rule.

Part 4

Following discussion in groups and with the instructor, use the rules discovered to find the errors involving exponents and make the corrections.

1. $(x^2)^3 = x^5$

1. Should use power to a power property and multiply the exponents so $(x^2)^3 = x^6$

2. $x^3 \bullet x^4 = x^{12}$

2. Multiplying with the same base so the exponents should be added. Write out in expanded form then simplify to one base if needed $x^3 \bullet x^4 = x^7$

3. $(2x)^3 = 2x^3$

3. The entire quantity of 2x is raised to the power of 3 so that would give $(2x)^3 = 2^3x^3 = 8x^3$

4. $(3y)^4 = 12y^4$

4. Similar to problem 3 raise the 3 and the y to the power of 4 $3^4 \neq 3 \cdot 4$ so $(3y)^4 = 3^4y^4 = 81y^4$

5. $\frac{w^6}{w^2} = w^3$ 5. Using the rule discovered in part 3 to divide with the same base

you should subtract the exponents not divide them

$$\frac{w^6}{w^2} = w^{6-2} = w^4$$

Part 5

Find the area of a rectangle with length of $(2zz^2)$ and a width of $(5xxy^3y^2)$

(Area = lw)

Multiply the 2 values for length and width but first write each in simplified form making it easier to apply the product property of exponents.

$(2zz^2) = 2z^3 \quad (5xxy^3y^2) = 5x^2y^5$

$$Area = l{\cdot}w = 2z^3 \cdot 5x^2y^5 = 10x^2y^5z^3$$

The area of a rectangle is $(27m^{12}p^6)$. If the length of the rectangle is $(3m^8p^3)$, find the width.

$Area = l{\cdot}w = (27m^{12}p^6) = (3m^8p^3){\cdot}w$

$\frac{(27m^{12}p^6)}{(3m^8p^3)} = w$ divide both sides of the equation by $(3m^8p^3)$

$(9m^4p^3) = w$

simplify using the exponential properties for dividing exponents with the a common base

Instructor Note: Although we used negative exponents in some examples, we did not include solutions with negative exponents. You could expand this ARC to include discovery of negative exponent properties.