TITLE OF LESSON: More on Domains of Rational Functions (Introducing Holes)

ESTIMATED TIME FOR LESSON (IN MINUTES): 40 minutes

SUGGESTED FORMAT (check all that are appropriate):

Individual in-class
Collaborative in-class
Individual homework
Collaborative homework

OVERVIEW:

Through guided explorations students will understand the relationships between factors of the numerator and denominator of a rational function and the function's zeros and its domain (including its "holes") both algebraically and graphically. This lesson focuses on rational functions with at least one common linear factor in the numerator and denominator.

PREREQUISITE IDEAS AND SKILLS:

- Definition of rational numbers
- Intervals and interval notation
- Definitions of function, the domain of a function, the range of a function
- Graphing functions on Cartesian coordinate systems
- Linear factors and zeros of a polynomial
- Factoring quadratics

MATERIALS NEEDED TO CARRY OUT LESSON:

- Access to Internet to show the Desmos graphs and some Geogebra applets given in the links inserted below
- Domains of Rational Functions Worksheet
- Domains of Rational Functions Answer Key

CONCEPTS TO BE LEARNED/APPLIED:

- How to divide one polynomial by another to form rational functions, where there is a common linear factor that is shared in each function's denominator and numerator.
- That the shared linear factor in a rational function's numerator and denominator (for each example used) relates to a hole in the graph of the rational function.

INSTRUCTIONAL PLAN:

Day 1: Setting the stage.

Briefly remind students that rational numbers are defined as having the form a/b where a and b are integers but $b \neq 0$. It is important for students to recognize that division by zero is not possible.

You might also need to provide a quick reminder to students on how to factor quadratic equations.

Have students solve the following in pairs:

Task 1. Let $f(x) = \frac{(x-1)}{(x-1)}$

a) Complete the table for the rational function $f(x) = \frac{(x-1)}{(x-1)}$.

x	$f(x) = \frac{(x-1)}{(x-1)}$
-3	
-2	
-1	
0	
1	
2	
3	

b) Use Desmos to graph the rational function: $f(x) = \frac{(x-1)}{(x-1)}$.

c) State the domain of the function.

In-class discussion

Once students have had a few minutes to complete the above, then ask the entire class "Is this function the same as g(x) = 1?" Typically at least one group will recognize that the two are not the same; instead, they vary by one point. If no groups point this out, have them create a table for g(x) using the same inputs they used for f(x). This should prompt conversation that you should lead to explain that the point (1, 1) is part of g(x) but not of f(x).

Next, as a class graph $f(x) = \frac{x}{x^3+6x^2+9x}$. Let students guide you through the process. It may be helpful to use a table as well as Demos to draw the graph. Be sure to label the hole on the graph.

In-Class Collaborative Worksheet

Have students work on the Domains of Rational Functions Worksheet in pairs or small groups. Circulate around to help as needed, especially if students struggle with factoring. The tasks are scaffolded to be increasingly difficult.

Some groups might struggle with the first tasks. Writing the domains will be challenging for the groups on Task 2, since it is in three parts. You may need to call the class to do Task 2 together as a group, if you see that the students are no longer engaging in productive struggle but seem to be overly frustrated.

You might want to wrap up the activity when at least a couple of groups have finished the fifth task.

From circulating about the group, select different individuals from the groups to go to the board and explain their group findings. If this is not part of your typical process, inform students prior to the activity that you will be doing this. If you have a classroom with boards around all the walls; it is a nice activity to have the groups complete at different sections of the boards.

Whole Class Discussion

Depending on how the different groups have done on the worksheet, you may need to provide additional examples to help them realize when rational functions have vertical asymptotes and when they have holes. Here are some that you could use.

$f(x) = \frac{x}{2x+8x^2}$ (has a hole when x = 0 and an asymptote at x = 4)

$f(x) = \frac{x-5}{x+3}$ (has an asymptote at x = -3)

$f(x) = \frac{x^2-3x-10}{x-5}$ (has a hole when x = 5)