TITLE OF LESSON: Categorizing Shapes and Functions by Symmetry

ESTIMATED TIME FOR LESSON (IN MINUTES): 45 minutes

SUGGESTED FORMAT (check all that are appropriate):

Individual in-class
Collaborative in-class
Individual homework
Collaborative homework

OVERVIEW:

Students explore different types of symmetry using letter tiles, then apply what they've learned to categorize graphs of functions by symmetry.

PREREQUISITE IDEAS AND SKILLS:

- Definition of a function; Vertical line test
- Familiarity with the Cartesian Coordinate System

MATERIALS NEEDED TO CARRY OUT LESSON:

- Copy of the *Letter Tiles Sheet* with a set of cut-out letter tiles for each group (or letter tiles sheets and scissors so they can cut their own).
- Copies of the *Even & Odd* worksheet for each group

CONCEPTS TO BE LEARNED/APPLIED:

- Types of symmetry
- Even and odd functions

INSTRUCTIONAL PLAN:

Goals

We will learn what it means graphically for a function to have "even" symmetry, "odd" symmetry, or neither.

Components

PART 1. Students manipulate letter tiles in order to familiarize themselves with three different types of symmetry.

PART 2. Students apply similar observations to graphs of functions to develop a graphical understanding of Even, Odd, and Neither.

Usage and Timing

This lesson is designed to be modular and flexible. You may use this as a stand-alone lesson or combine it with other lessons in the same ARC. Parts of this lesson may also be used

independently. You may wish to give students more or less than the suggested amount of time for each activity depending on how thoroughly you wish to explore the topic.

PART I - Introduction to Symmetry Using Letters

Preparation (before class)

Print and cut out the provided letter tiles from the *Letter Tiles Sheet*, one set for each student group. Make sure the letters can be seen through the back of the paper when held up to the light.

Activity and Discussion (20 minutes)

- Split into small groups.
- Give each group a set of letter tiles. Have them cut the tiles apart, if necessary.
- Ask students to find the H tile.
- Show that if you hold the paper up to the light you can see the letter H through the back.
- Ask in what ways can you manipulate the tile and still see the letter H in its proper orientation?
- Take student input until at least three answers are given (in no particular order): 1. Flip the tile over sideways, 2. Flip the tile over top to bottom, 3. Turn the tile upside down (or rotate the tile). (They may also include "do nothing").
- Introduce the word *symmetry*, stating that an object has symmetry if you can change its orientation (move it around) and it still looks the same. Make the point that each way to move the tile so that you still see an H is a different type of symmetry.
- Give the following instructions and let students work independently or in groups to categorize their tiles by symmetry.

Group Work (10 minutes)

- Your goal is to sort your tiles according to their symmetries; find and name at least three categories of symmetry (there may be more). Remember there were three different types of symmetry for the letter H.
- Name each category in a way that makes sense to you.
- Under each category name, write down the letters that belong in that category.
- Hint: A given letter may belong to more than one category.

Possible Responses (note that groups may have used their own terminology for each)

- Symmetry Over y-axis: A H I M O T U V W X Y
- Symmetry Over x-axis: B C D E H I K O X
- Symmetry About the Origin (or one axis then the other, or rotational symmetry): H I N O S X Z
- Both Symmetry Over y-Axis and Symmetry Over x-axis: H I O X
- No Symmetry: G J L Q R

Follow-Up Discussion (10 minutes)

- Collect results on the board or projector where everyone can see (optionally, this may be done during the activity).
- If any student categories are NOT based on symmetry, discuss what it would take to be able to call them symmetry-based categories. Use manipulation of the letter tiles as needed.
- Acknowledge student-provided category names and give "Official" names to the different symmetries according to your own curriculum.
- Draw representative letters on a coordinate axis system so that terms like "symmetry over the y axis" will have context.
- Announce that we are turning our attention to functions, and introduce the terms *Even* and *Odd*. Even functions have symmetry over the y-axis, and odd functions have symmetry over both axes, or in other words, about the origin.
- Note that not all of the letter shapes could represent functions, as seen using the vertical line test. The descriptors "Even" and "Odd" should only be applied to functions.
- "Bonus" question: Which letters might represent Even or Odd function shapes?
    - Even: V W (Note that M and U would not be functions, in the font used here, since they wouldn't pass the vertical line test.)
    - Odd: None (Note that N would not be a function, in the font used here, since it wouldn't pass the vertical line test.)

PART 2 - Symmetry in Functions

Preparation (before class)
Print copies of the provided *Even & Odd* worksheet for each class member.

Activity and Discussion (25 minutes)

Group Work (10 minutes)

- Have students work in small groups to complete the worksheet, labeling each function as even, odd, neither, or both.
- Ask them to come to consensus on their answers and explain their thinking to one another as needed.
- Let them know you'll have some challenging questions for them to discuss when they finish.

Follow-Up Discussion (15 minutes)

- Make sure everyone is correctly identifying even, odd, and neither. For any miscategorized functions, use the definitions (symmetry over the y-axis / symmetry over the origin) to help students re-evaluate.

- Collect student-provided examples of even, odd, neither, or both functions from problems 19 - 24 on the worksheet. Share these anonymously on the board or projector where everyone can see (may be done during the activity).
- As a class, evaluate whether all student-provided examples are correctly categorized and move them as needed. Emphasize that an incorrect answer provides a valuable opportunity to learn and discuss. When a student identifies a wrongly placed function, make sure they also give a reason for their assertion.
- Challenge Questions:
    - Why are there no functions on the worksheet with symmetry over the x axis? / Was anyone able to draw one?
    - Why are there no functions on the worksheet that are both even and odd? / Does such a function exist?
- Answers to the Challenge Questions:

  Student answers may vary; the point is to get them really thinking about it. But in general, if a graph has symmetry over the x-axis, it won't pass the vertical line test. Neither will a graph that is both even and odd. However, there is one notable exception: the function $f(x) = 0$ (or any version of $f(x) = 0$ with a symmetrically restricted domain) is symmetric over the x-axis and is both even and odd.