Painter’s Assistant

This lesson reinforces the idea of a rational function with a real-world situation. It is designed to highlight the concept of a horizontal asymptote as a limiting value as the independent variable approaches infinity.

**Guiding Principles**

1. Active Learning: Students will work together to find a solution to a problem that requires them to seek out or select the information required, perform calculations, and evaluate their actions in the context of the problem.
2. Meaningful Applications: Students will work on an interesting application with perhaps multiple solution paths, where they will identify a mathematical function that models the situation.
3. Academic Success Skills: Students use intuition and perseverance to recognize that they can find solutions to real-life problems.

**Prerequisite Knowledge**

1. Sketching graphs
2. Solving Rational Equations

**Objectives**

1. Students will demonstrate knowledge concerning practical interpretation of horizontal asymptotes.
2. Students will be able to choose a shape for the graph that fits their expectations.
3. (Optional) Students will be able to write a rational function that models given data

**Materials**

The PowerPoint, a blank sheet of paper for each student, and a graphing tool (Desmos, a graphing calculator, or similar graphing device).

Teacher’s Guide (~ 45 minutes)

1. Slide 1: Cover slide. Each student should have a blank sheet of paper to take notes and do calculations.

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1. Slide 2: Sets up the problem by giving some background information

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1. Slide 3: Continues to set up the problem by giving more relevant information. And introduces the main question for this problem.

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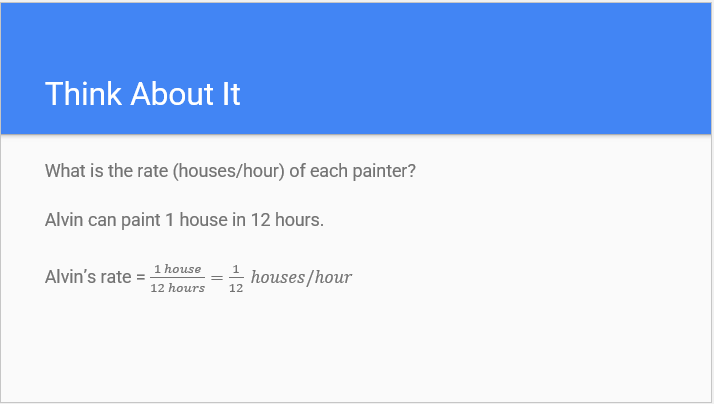
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1. This slide prompts the students to think carefully about an important variable in the problem: the rate at which each painter can paint a home in (houses/hour). Note, that since no painter can paint an entire house in one hour or less, the answers will be fractional. For instance, Alvin’s rate to paint is 1/12 of a house per hour.

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1. Slide 5: This slide gives the answer to questions in the previous slide.



1. Slide 6: This slide continues to build off the other slides adding in the next step of complexity: working together.

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1. Slide 7: This slide prefaces the main question (slide 3) by asking how much work two painters can get done in a certain amount of hours. Students should try to arrive at the equation:

To help them arrive at this equation, you can ask them to analyze the units

,

where the unit is.

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1. Slide 8: Introduces a question that gets to the heart of the problem. How many hours, , would it take Alvin with an assistant painter who paints at a rate of , to paint one entire house. Write as a function of (the number of hours it takes to paint one house). Students may need help finding the formula: rate × time = 1 (house), where the time is *h* hours, so (1/12 + 1/*x*)·*h* = 1, or alternatively 1/12 + 1/*x* = 1/*h*. Solving for *h* gives the required function, .

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1. Slide 9: Gives a link to the Desmos graph of the problem, <https://www.desmos.com/calculator/crbkc57dmu>. Students can use the slider to visualize. Ask students what the *y*-axis represents. Ask what the *x*-axis represents. Move the slider for the input variable to get points on the graph. Ask students what each point means.

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1. Slide 10: Revisits and answers the main question on slide 3. Be careful with the answer to “12 hours?” It’s a trick question: Al can do it by himself in 12 hours!

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**Common Student Pitfalls**

1. The idea of the combined rate may be difficult for students. It may help to draw a picture of a simplified situation. Suppose one painter can paint a tiny house in 2 hours and another can paint it in 4 hours. So, in one hour the first painter paints ½ the house and the second painter paints ¼ of the house. Together, they can paint ¾ of the house in one hour: ½ + ¼ = ¾ . Setting up the equation ½ + ¼ = 1/*h* yields *h* = 4/3 hours to paint the entire house.

Remind students that Alvin’s rate is 1/12 houses per hour and his assistant’s rate is 1/*x* houses per hour, so their combined rate is 1/12 + 1/*x* houses per hour.