1. What does $f'(x)$ represent in terms of the graph of a function f?

2. Suppose you are given two points, $(a, f(a))$ and $(b, f(b))$, on a graph of the function f. How would you find the slope of the secant line through those points?

3. Consider the function $f(x) = x^3 + 2x^2 - x - 2$.

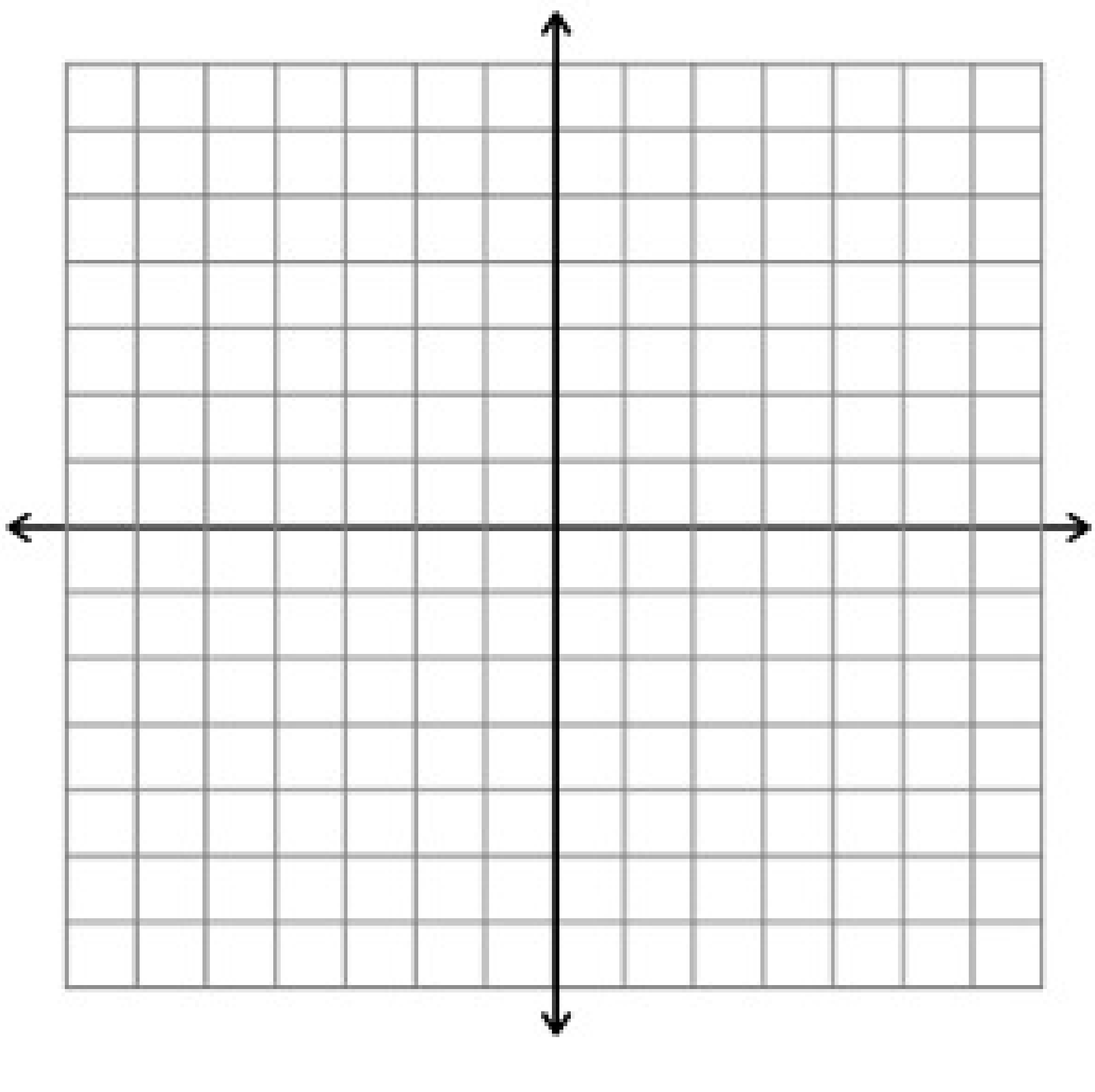
   (a) Graph f on $[-2, 1]$ on the plane given on page 2.

   (b) Describe the graph of f on $[-2, 1]$. List as many characteristics as possible.

(c) Graph the secant line through $(-2, f(-2))$ and $(1, f(1))$ on the same plane.

(d) What is the slope of the secant line?

(e) Is there a c in the interval $(-2, 1)$ such that $f'(c)$ is equal to the slope of the secant line from part (c)? Why or why not?



4. Consider the function $g(x) = \frac{1}{x^2}$.

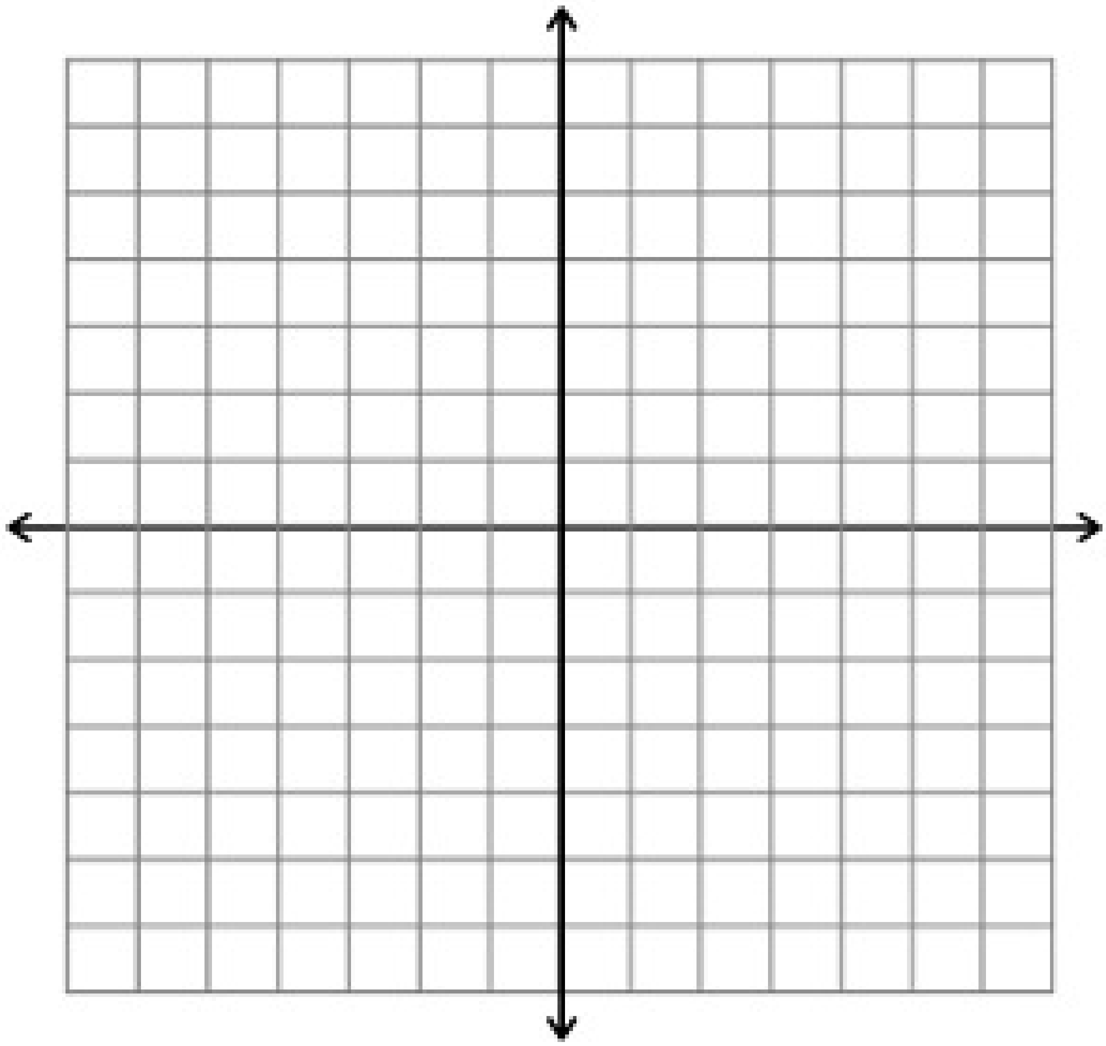
   (a) Graph g on $[-2, 2]$ on the plane given on page 4.

   (b) Describe the graph of g on $[-2, 2]$. List as many characteristics as possible.

   (c) Graph the secant line through $(-2, g(-2))$ and $(2, g(2))$ on the same plane.

   (d) What is the slope of the secant line?

   (e) Is there a c in $(-2, 2)$ such that $g'(c)$ is equal to the slope of the secant line from part (c)? Why or why not?



5. You previously listed physical characteristics of $f(x) = x^3 + 2x^2 - x - 2$ and $g(x) = \frac{1}{x^2}$.

   (a) How are the functions similar?

(b) How are the functions different?

6. Now, consider the following theorem:

Theorem 1 (Mean Value Theorem) *Suppose* $y = h(x)$ *is a continuous function on a closed interval* $[a, b]$ *and differentiable on the interval's interior* (a, b)*. Then there is at least one point* c *in* (a, b) *at which*

$$h'(c) = \frac{h(b)-h(a)}{b-a}.$$

(a) What does $h'(c)$ represent in terms of a graph?

(b) What does $\frac{h(b)-h(a)}{b-a}$ represent in terms of a graph?

(c) What does $h'(c) = \frac{h(b)-h(a)}{b-a}$ mean graphically?

(d) Based on the Mean Value Theorem, why did f (from problem 3) have a tangent line with the same slope as the secant line but g (from problem 4) did not?

(e) Graph a random function f which is continuous on the closed interval $[-3, 2]$ and differentiable on the open interval $(-3, 2)$. Graph the secant line through $(-3, f(-3))$ and $(2, f(2))$. Find an x in the interval $(-3, 2)$ such that the tangent line at x is parallel to the secant line. Graph that tangent line.

7. A consequence of the Mean Value Theorem is Rolle's Theorem.

Theorem 2 (Rolle's Theorem) *Suppose $y = h(x)$ is a continuous function on a closed interval $[a, b]$ and differentiable on the interval's interior (a, b). If $h(a) = h(b)$, then there is at least one number c in (a, b) at which*

$$h'(c) = 0.$$

(a) Why is Rolle's Theorem a consequence of the Mean Value Theorem?

(b) [1] The function

$$f(x) = \begin{cases} 0, & x = 0 \\ 1 - x, & 0 < x \leq 1 \end{cases}$$

is zero at $x = 0$ and $x = 1$ and differentiable on $(0, 1)$, but its derivative on $(0, 1)$ is never zero. How can this be? Doesn't Rolle's Theorem say the derivative has to be zero somewhere in $(0, 1)$? Justify your answer.

[1] This question is based off of a problem found in Thomas' Calculus, 13th Edition, section 4.2, problem 13.

(c) Give an example of a function f that is continuous on a closed interval $[a, b]$ (be sure to include the interval) such that $f(a) = f(b)$ but f is not differentiable on the open interval (a, b). Using the function you created and the corresponding interval, does there exist a c in (a, b) such that $f'(c) = 0$? Does Rolle's Theorem apply? Why or why not?

(d) Give an example of a non-constant function f and domain which satisfies the assumptions of Rolle's Theorem. Then, find a value of c in your domain such that $f'(c) = 0$.