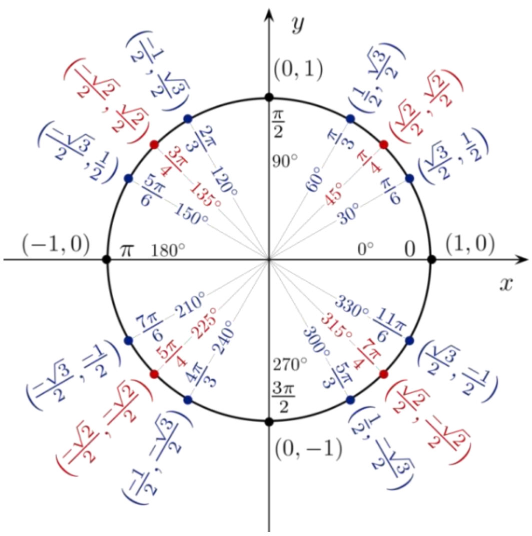
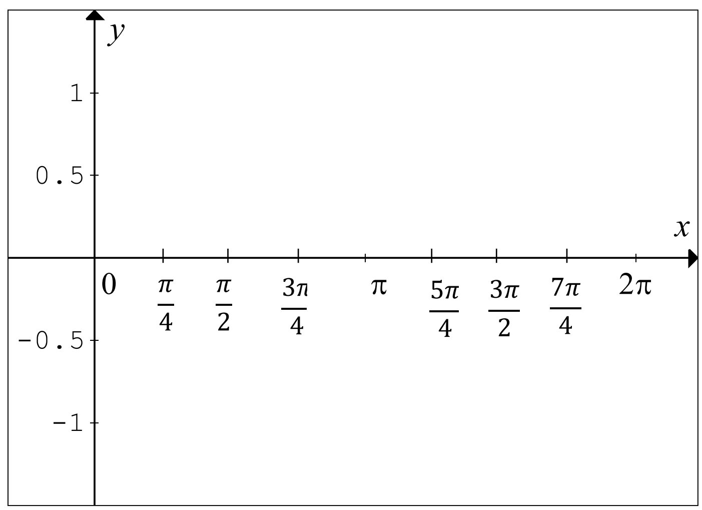
# Introduction to Sine and Cosine Graphs

1. We're going to use the unit circle to plot the graph of $y = \sin(x)$ on a rectangular coordinate grid. To start, we'll lay the radian angles out horizontally on the $x$-axis with 0 radians at the origin and positive angles to the right, with negative angles to the left.

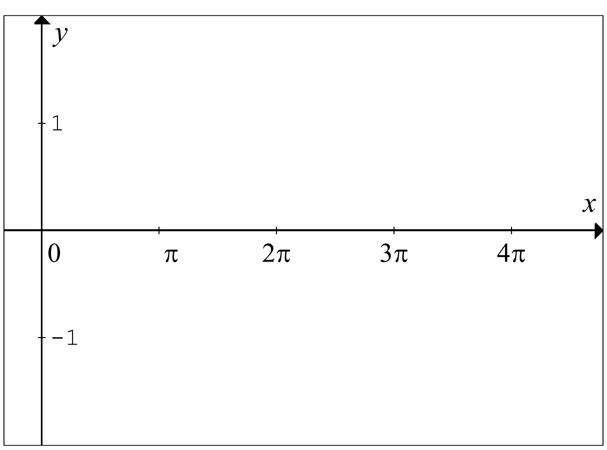
Start by circling all the *sine* values (the $y$-values on the unit circle) of points on the unit circle.

Next, plot points on the rectangular grid so that the first coordinate is the angle (in radians) and the second coordinate is the sine value. Plot points for the multiples of $\frac{\pi}{4}$ angles between 0 and 2π.



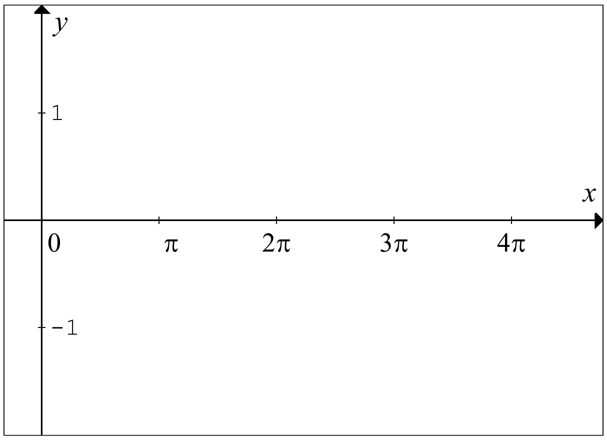




2. Now go to [https://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html](https://www.mathsisfun.com/algebra/trig-interactive-unit-circle.html) . This interactive website allows you to move your cursor around a unit circle. Use the values for the sine function (shown in green) as you move the cursor from 0 to 2π radians to plot the points for the sine function. Then repeat it for values between 2π and 4π.



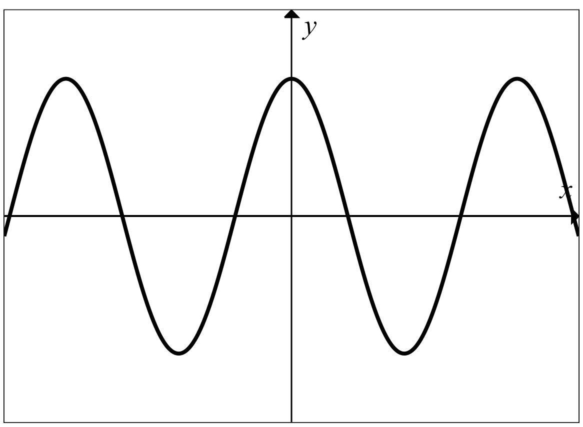

*What do you notice about the graph? List your observations here:*

3. Repeat what you did above using the interactive website, but this time use the values for the cosine function as you move the cursor from 0 to 2π radians to plot the points. Then repeat it for values between 2π and 4π.

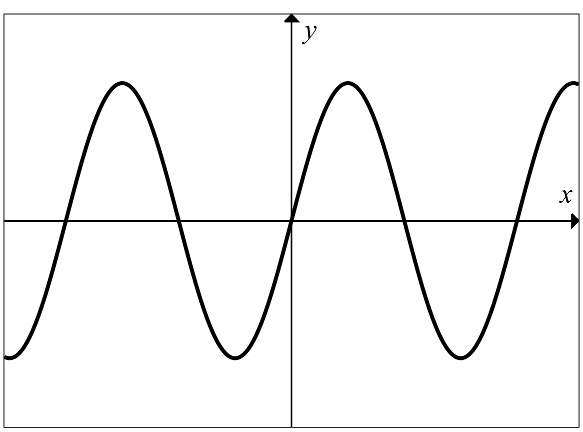



*What do you notice about the graph? List your observations here:*

4. For the two graphs below, determine which is the graph of $y = \sin(x)$ and which is the graph of $y = \cos(x)$. Mark the ***x*-intercepts** and the ***x*-coordinates that correspond to the highest and lowest *y*-values.** Mark the ***y*-axis with the highest and lowest values.**

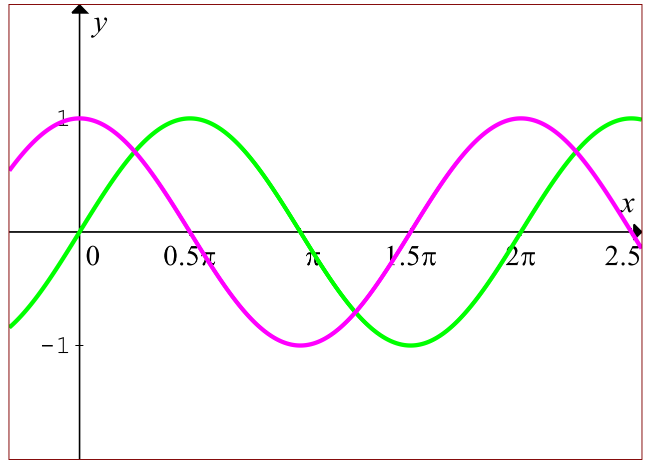



Graph: *y* = ____________




Graph: *y* = ____________

5. The **period** of a trigonometric function is the length of one complete cycle. This is the smallest positive value in which the function repeats.




What is the period of $y = \sin(x)$?

What is the period of $y = \cos(x)$?

6. The **amplitude** of a trigonometric function is half the distance between the lowest and highest points.

What is the amplitude of $y = \sin(x)$?

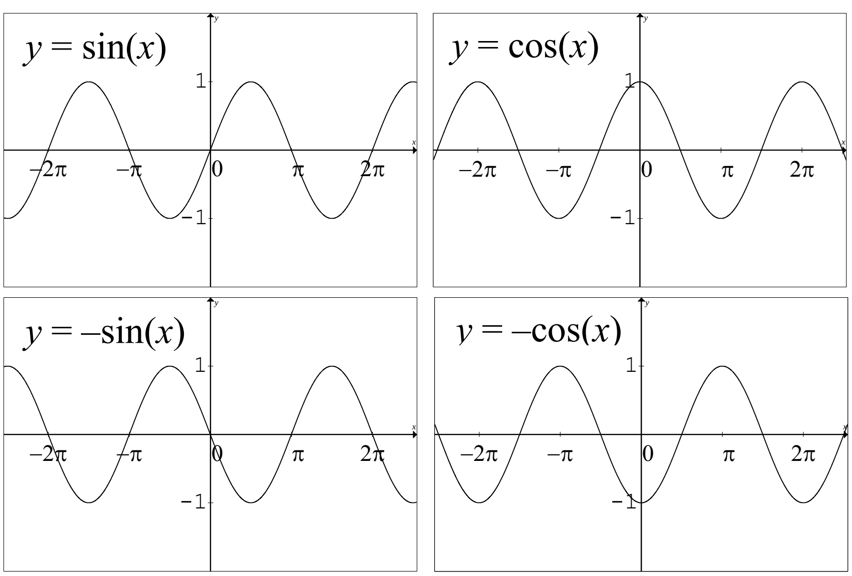
What is the amplitude of $y = \cos(x)$?

7. The **midline** of a trigonometric function is the horizontal line halfway between the lowest and highest points.

What is the midline for $y = \sin(x)$?

What is the midline for $y = \cos(x)$?

8. We can apply transformations to the sine and cosine graphs in the same way we apply them for other types of functions. To start, compare the graphs below so see how multiplying by –1 affects each graph.




← Basic sine and cosine graphs

*How do these graphs compare to their "parent" functions above?*

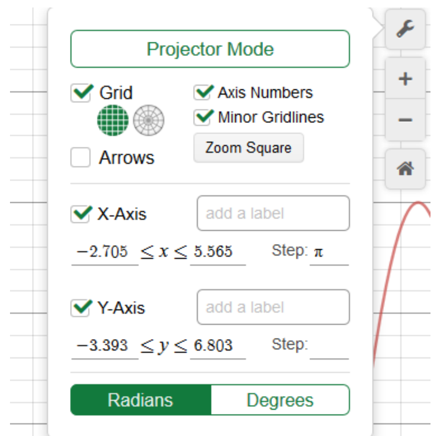
9. We'll use a Desmos graphing calculator to explore them.

Go to: https://www.desmos.com and choose Graphing Calculator.

On the line 1, type the function $y = Asin(Bx) + k$.

Add sliders for all three of the constants $A$, $B$, and $k$. Click on the slider for $B$ to change it so that $0.25 \leq B \leq 10$, with step 0.25. Change the step size for $A$ and $k$ to 1. All three constants can be varied by using the sliders. To get units of π, click the wrench icon in the upper right corner and change the "Step" to π for the $x$-axis.

- **Zoom in or out**: Use the + or – in the upper right corner, or by using the scrolling wheel on your mouse.
- **Change the viewing window**: Click on the wrench symbol in the upper right corner and change the minimum or maximum values for either the $x$-axis or the $y$-axis.




10. First, move the sliders to $B = 1$, $k = 0$, and $A = 1$ to graph $y = \sin(x)$.

- Then, slide $k$ to different positions to see what it does to the graph.

  *How does changing k affect the graph?*

- Now slide $k$ back to 0 and slide $A$ to different positions.

  *How does changing A affect the graph?*

- Now slide $A$ back to 1 and slide $B$ to different positions.

  *How does changing B affect the graph?*

11. Move the sliders to appropriate positions to graph $f(x) = -2\sin(x) + 3$. Then find each of the following.

Period = _______ Amplitude = _______ Vertical Shift: _______ units up/down (circle one)

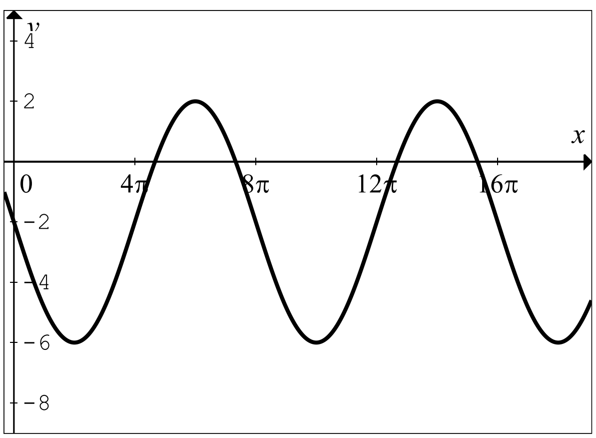
What is the midline for this graph? $y$ = _____

12. Change the "sin" on line 1 to "cos" and then move the sliders to appropriate positions to graph $g(x) = 5\cos\left(\frac{1}{2}x\right) - 2$. Then find each of the following.

Period = _______ Amplitude = _______ Vertical Shift: _______ units up/down (circle one)

What is the midline for this graph? $y$ = _____

13. Move sliders to find the $A$, $B$, and $k$ values necessary to produce the graph below.




Write your answers here:

A = _______

B = _______

k = ______

What is the midline for this graph?

$y$ = ________

14. Use Desmos to help you answer this True/False question. Assume $A, B \neq 0$.

**True or False?** It is possible for the graph of a cosine function, $y = A\cos(Bx) + k$, to pass through the point (0, 0). If true, what A, B, and k values can you use? If false, explain why.

**Summary**

Equations of the form $y = A\sin\big(B(x-h)\big) + k$ or $y = A\cos\big(B(x-h)\big) + k$ have:

*Amplitude*: $|A|$ *Period*: $P = \frac{2\pi}{|B|}$ *Vertical shift*: $k$ *Horizontal shift*: $h$

Note: The horizontal shift of $h$ units is called the ***phase shift.***

If $A < 0$, there is an $x$-axis reflection. If $B < 0$, there is a $y$-axis reflection.

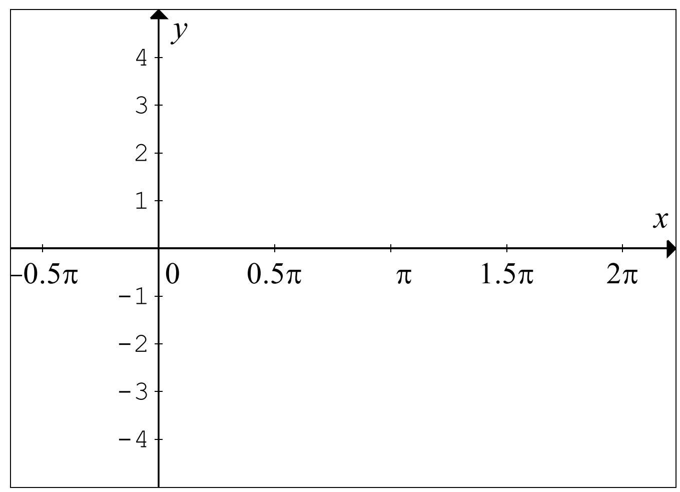
15. For the function $f(x) = 3\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$, find each of the following.

(a) Amplitude

(b) Period

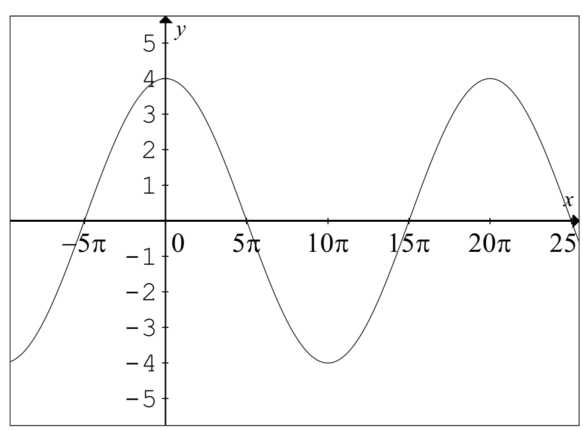
(c) Phase shift

(d) Sketch the graph.




16. A transformed cosine function is given. Find each of the following.

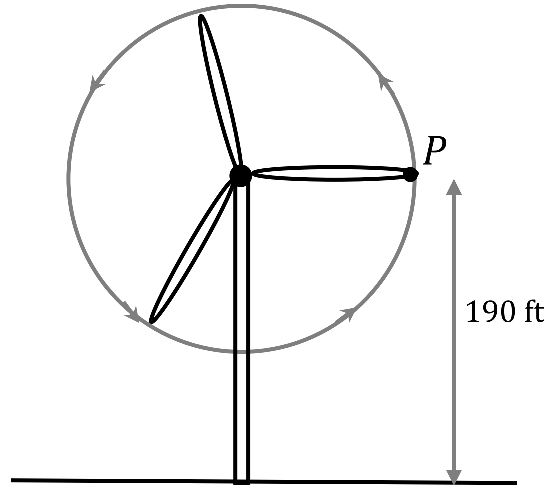
(a) Describe a set of transformations that could relate this graph to the graph of $y = \cos(t)$.




(b) Write a formula for this graph as a **cosine** function.

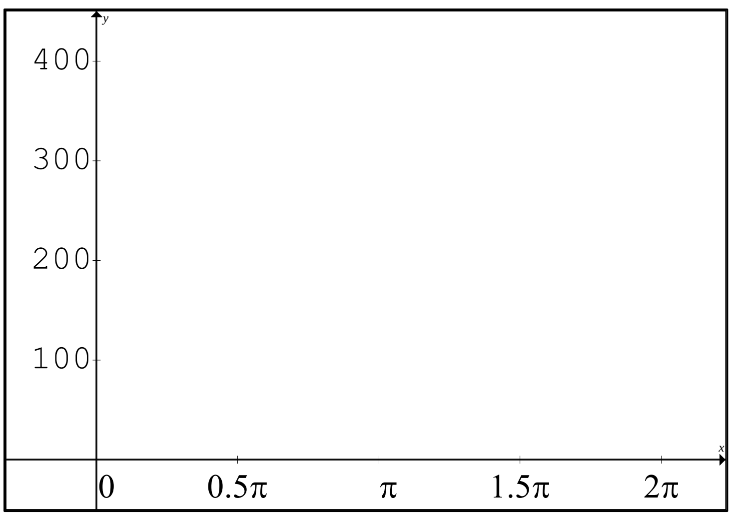
(c) Write a formula for this graph as a **sine** function.

17. A wind turbine has blades that are 110 feet long, attached to a tower that is 190 feet tall. Assume the blades rotate counterclockwise and that point $P$ is located at the tip of one of the blades at the 3 o'clock position when the wind turbine starts rotating.




(a) What are the highest and lowest points reached on the wind turbine?

(b) Graph the height function and find the amplitude and midline.




(c) Find the amplitude, period, and midline.

(d) Find a formula for the height of the point $P$ above ground level, as a function of the angle $\theta$ radians.

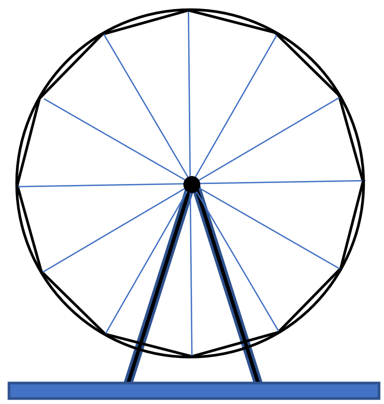
18. Suppose a Ferris wheel that is 300 feet in diameter completes one rotation every 20 minutes. The boarding platform is 5 feet above the ground. Assume a rider boards the wheel at its lowest position (the 6 o'clock position), and the wheel turns in the counterclockwise direction. The graph represents the rider's height above the ground, in feet, $t$ minutes after the boarding the ride. Determine each of the following.

(a) Find the period, amplitude, and midline.

Period: ________

Amplitude: ________

Midline: $y$ = ________



(b) The graph that models the Ferris wheel's height at time $t$ minutes is shown below. You'll need to model the situation with either sine or cosine. Which should you use?

$y = A\sin\big(B(x-h)\big) + k$ or $y = A\cos\big(B(x-h)\big) + k$.

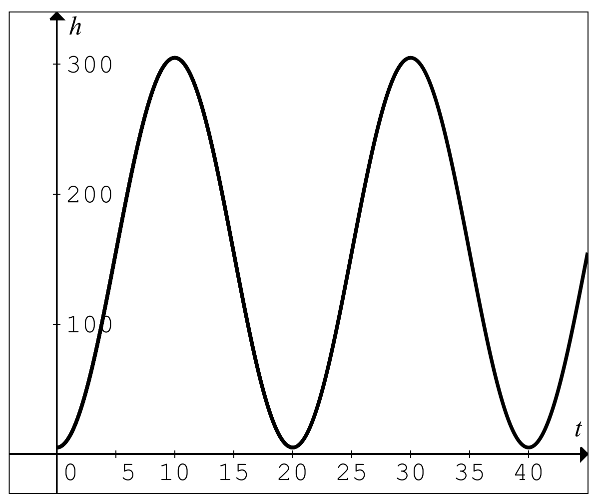
For the function you chose to use, what values of $A$, $B$, $h$, and $k$ are necessary?

$A$ = _______

$B$ = _______

$h$ = _______

$k$ = _______




Write a formula that represents the rider's height $h$ above the ground, in feet, as a function of $t$ minutes after boarding the ride.

Formula:

$h = f(t) =$ ________________________________