Completing the Square Activity

Today, we'll use "algebra tiles" and teamwork to explore the concept of "Completing the Square," an important algebraic technique involving quadratic expressions.

Part 1: Exploring with Algebra Tiles

1. Experiment 1: Arrange 1 large square (x^2) and 6 long rectangles (x) to start forming a larger square. How many small squares (1×1) are needed to complete the square?

   Answer: 9

2. Experiment 2: Now try with 1 large square (x^2) and 8 long rectangles (x). This time, how many small squares are needed?

   Answer: 16

3. Table Activity: In your group, experiment with different numbers of long rectangles (always using even numbers) and record your finding in the table below:

# of Long Rectangles (x)	# of Small Squares Needed
2	1
4	4
6	9
8	16
10	25
12	36

4. Discussion Question: Do you notice any pattern or relationship between the number of long rectangles and the number of small squares needed? Describe your observations.

   To get the number of small squares needed, we first take half the number of long rectangles and then square it. That is:

   # of small squares = [(1/2) · (# of long rectangles)]^2

Part 2: Understanding Perfect Square Trinomials

Objective: Relate your findings from Part 1 to algebraic expressions and the concept of perfect square trinomials.

1. Reflection: In Part 1, you found that if you have 6 long rectangles, then 9 small squares are needed to complete the square. By considering two ways of finding the area of the large square that you formed, explain (without using algebra) how this shows that $x^2 + 6x + 9 = (x+3)^2$.

   The area of the large square can be found in two ways:

   (1) by summing the areas of the individual tiles, which gives $x^2 + 6x + 9$, or
   (2) by multiplying the length of the square by its width, which gives $(x+3)^2$.

   Since these are two different ways of calculating the same thing (the area of the large square), they must be equal.

2. Challenge Problem: Given the expression $x^2 + 8x + 11$, use your algebra tiles to attempt to form a larger square. Will there be any tiles missing or extra? How many?

   There are 5 missing small tiles.

3. Algebraic Representation: Based on the challenge problem, represent $x^2 + 8x + 11$ as the square of a binomial plus or minus a constant. That is, rewrite it in the form $(x \pm h)^2 \pm k$.

   We have:

   $$x^2 + 8x + 11 + 5 = (x+4)^2.$$

   So, subtracting 5 from both sides:

   $$x^2 + 8x + 11 = (x+4)^2 - 5.$$

Part 3: Completing the Square with Equations

Objective: Use the technique of completing the square to solve quadratic equations.

1. Equation 1: Solve $x^2 + 10x + 14 = 0$ using the method of completing the square.

   $x^2 + 10x = -14$ (subtract 14 from both sides)

   $x^2 + 10x + 25 = 25 - 14$ (add 25 to both sides, so we have a p.s.t on the left side)

   $(x+5)^2 = 11$ (factor the p.s.t. on the left side and compute 25-14=11)

   $x + 5 = \pm\sqrt{11}$ (take the square root of both sides, remember the $\pm$)

   $x = -5 \pm \sqrt{11}$ (subtract 5 from both sides)

2. Equation 2: Solve $x^2 - 6x + 5 = 0$ using the method of completing the square. Then use factoring to confirm your solution.

Completing the Square solution:

$x^2 - 6x = -5$

$x^2 - 6x + 9 = 9 - 5$

$(x - 3)^2 = 4$

$x - 3 = \pm 2$

$x = 3 \pm 2$

$x = 5, 1$

Factoring solution:

$x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$

$x = 5, 1$

Part 4: Application Challenges

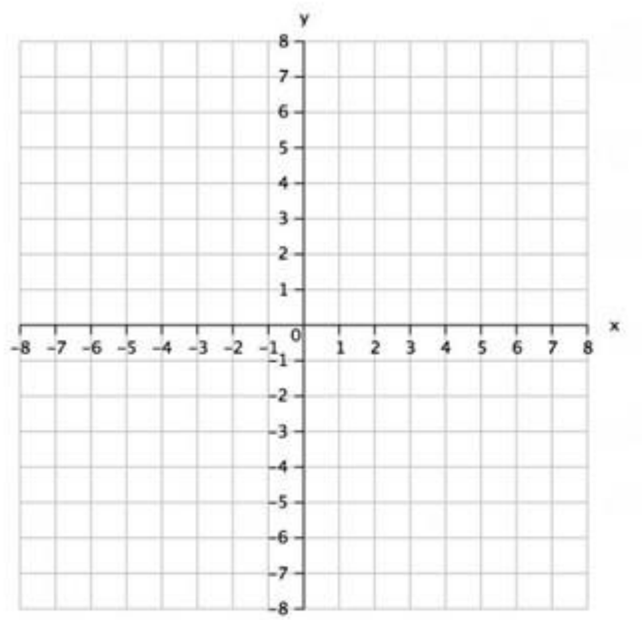
Objective: Apply completing the square to various scenarios.

1. Graph a Parabola: Sketch the graph of the parabola $y = x^2 - 6x + 8$ by using completing the square along with transformations of the graph of $y = x^2$. (Do not use the axis of symmetry or a table of values.) Then state the vertex of the parabola.

$y = x^2 - 6x + 8$

$y = x^2 - 6x + 9 - 9 + 8$

$y = (x + 3)^2 - 1$



Vertex: (-3, 1)

2. Center and Radius of a Circle: Determine the center and radius of the circle given by the equation $x^2 + 4x + y^2 + 6y - 12 = 0$ by using completing the square.

$$x^2 + 4x + y^2 + 6y - 12 = 0$$

$$(x^2 + 4x) + (y^2 + 6y) = 12$$

$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$$

$$(x + 2)^2 + (y + 3)^2 = 25$$

Center: (-2,-3) Radius: 5

Part 5: Class Discussion to Derive the Quadratic Formula

You've probably seen the Quadratic Formula:

The solutions to $ax^2 + bx + c = 0$, where $a \neq 0$, are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Where does this mysterious formula come from? Follow along with a class discussion to see.