The derivative at a point, Part 2 – Instructor notes

The purpose of this activity is for students to begin to generalize their ideas of derivative previously developed in a familiar context of distance-time-velocity to other rate of change contexts. This should support students in moving away from thinking of derivative as modeling merely speed or slope of a tangent to thinking of derivative as modeling instantaneous rate of change more generally.

The half-life of Iodine-123, used in medical radiation treatments, is about 13.2 hours. Approximate the instantaneous rate at which the Iodine-123 is decaying 3 hours after a dose of 8.4 μg is administered accurate to within 0.0001 μg/hr.

Don't let students spend too much time determining the exact exponential equation for the amount of iodine. Although this is important, this is not the focus of this lab. Feel free to quickly give students the formula $I = 8.4e^{-\frac{\ln 2}{13.2}t}$ or $I = 8.4\left(\frac{1}{2}\right)^{t/13.2}$ so they can focus on the more important tasks of reasoning about rates of change.

Encourage students to be creative in drawing a diagram so that they can illustrate changes. For example, previous classes have drawn a figure of a person "filled" to different levels with radioactive material. Some have drawn simpler pie charts or mounds or test tubes of radioactive iodine. In order to get within 0.0001 μg/hr, students will need to use $-0.01 \text{ hr} < \Delta t < 0.01 \text{ hr}$.

Help students reason that the rate is increasing (note that $I'(t)$ is negative and increasing toward 0), ask them whether there is more change in the amount of radioactive iodine in the first 13.2 hours or the second 13.2 hours.

Instantaneous rate = –0.3768 μg/hr

Underestimate rate = –0.3769 μg/hr

Overestimate rate = –0.3767 μg/hr