The derivative at a point, Part 1

Preparation: A bolt is fired from a crossbow straight up into the air with an initial velocity of 49 m/s. Accounting for wind resistance proportional to the speed of the bolt, its height above the ground is given by the equation $h(t) = 7350 - 245t - 7350e^{-t/25}$ meters (with t measured in seconds). Throughout this activity, you will approximate the speed when $t = 2$ seconds.

A. Draw a *full-page* picture of the physical context in three different configurations of the crossbow bolt *overlayed*. This will help illustrate how the two quantities are changing near the time of interest $t = 2$ seconds. You will redraw and add to this picture during class.

B. What happens to the changes in the height as the time increases by constant amounts? Is the rate of change (speed) constant, increasing or decreasing?

C. Draw a *full-page* graph showing the relationship between the two quantities involved in the instantaneous rate that you are asked to approximate. Add a point for each of the configurations you drew in your picture from A. Represent the changes in both quantities on your graphs as the length of short line segments. You will redraw and add to this graph during class.

D. On your picture *and* your graph illustrate and label the *changes* in height and *changes* in time to support your answer to B (using both Δ-notation and numerical values).

The derivative at a point, Part 1

Class Activity: A bolt is fired from a crossbow straight up into the air with an initial velocity of 49 m/s. Accounting for wind resistance proportional to the speed of the bolt, its height above the ground is given by the equation $h(t) = 7350 - 245t - 7350e^{-t/25}$ meters (with t measured in seconds). Throughout this activity, you will approximate the speed when $t = 2$ seconds.

1. Draw a *full-page* picture of several "snapshots" showing
   a. The bolt at time $t = 2$ seconds.
   b. The bolt at other times from your preparation work.
   c. Changes in height and time (using both Δ-notation and numerical values) to support your answers in the preparation.

2. Draw a *full-page* graph showing
   a. Several points corresponding to the bolt at time $t = 2$ seconds and three or four nearby times.
   b. Algebraic and numerical representations of the height and time at the points in Part a
   c. Changes in height and changes in time starting from time $t = 2$ seconds (using both Δ-notation and numerical values)

3. In this question, you will provide details about *what you have been asked to approximate*.
   a. Describe what you have been asked to approximate using language about the crossbow bolt.

   b. Define a variable to represent this unknown value algebraically. What units will be attached to it?

   c. Represent this unknown value that you are approximating on your graph. Label it with your chosen variable. What attribute of the object that you added to your graph corresponds to the unknown value you are approximating?

4. In this question, you will provide details about *approximations* to your instantaneous speed.
   a. Compute 3 average rates of change that approximate the speed of the crossbow bolt at time $t = 2$ seconds.

   b. Using language about the crossbow bolt, explain the physical meaning of one of your average rates of change.

   c. Write an algebraic expression showing someone how to compute these average rates in general.

   d. Represent the 3 approximations on your graph. Label them with numerical values. What attribute of the things that you added to your graph corresponds to the approximation values?

5. In this question, you will identify both *underestimates* and *overestimates* for the requested instantaneous rate.

   a. If you have not already found both underestimates and overestimates and represented them on your graph, do so.

   b. Using only language about your physical context (not your graph), explain how you know these are in fact underestimates and overestimates.

   c. Explain how your explanation from Part b can be seen on both the picture of the situation and on the graph.

6. In this question, you will identify and represent the *errors* in your approximations.
   a. Give an algebraic representation of the errors for both an underestimate and an overestimate.

   b. Explain how these errors are represented graphically. Add and label the errors on your graph.

7. In this question, you will identify and represent the *error bounds* in your approximations.
   a. Find an error bound for one of your approximations. Justify your answer.

   b. Explain how this error bound is represented graphically. Add and label the error bound on your graph.

   c. What is the resulting range of possible values for your instantaneous rate? Explain how this range is represented graphically.

8. Find an approximation accurate to within the error bound given in your problem. Show and explain all of your work.