Mean Value Theorem Introduction Activity

Instructor Notes:
Note that this is meant to be the first activity that students see to motivate the Mean Value Theorem. The Mean Value Theorem is not mentioned at all; however, the key concepts needed to understand it are.

Active Learning:
This activity is meant to engage students as active participants in their own learning through an emphasis on communication and reflection.

Meaningful Application:
This activity leverages a real-life scenario that should resonate with students allowing them a common ground to identify different rates of change to investigate the claims that are made.

Student Academic Success:
The activity has a low floor, allowing all students to productively engage in the activity based on their personal experiences.

Below, the instructions for the instructor are given in red. Any side comments to provide additional information will be in brackets. Any information or prompts that should be shared with the students are in black. Note that groups may move at different speeds; so, the prompts given are an attempt to try to keep all students at roughly the same pace.

Activity:
[At the very beginning of class, even before I share the scenario, I let the students know that each of them will have to turn in their work for a grade. I find that this motivates students to not try to coast in the collaborative group work that comes with this activity.]

The following scenario is presented to students as a whole class. [I usually project this and leave it up for the entirety of the activity. I also ask if anyone has any questions and then discuss, as needed, to make sure that all understand the scenario presented.]

The police monitor a long, straight stretch of road where the speed limit is 65 mph. They have checkpoints 6 miles south of town and 21 miles south of town. A police officer sees a car pass through the first checkpoint at 2:04 pm, and another officer sees the same car pass the second checkpoint at 2:14 pm. The car's speed at the first checkpoint is 60 mph and is 63 mph at the second checkpoint.

The officer gives the driver a speeding ticket, and the driver tries to argue that he was not speeding. Formulate an argument (including a visual aid) that the officer could use in court to show that the driver MUST have been speeding.

After the students have had a couple of minutes to work on this individually, have them find a partner to work on this. Circulate around the room to gauge how students are doing. After a few minutes give the class the following hints to be sure they are on the right track.

Does this scenario represent a function? If so, then determine what the possible inputs and outputs would be.

After the partners have had a couple more minutes to work share these hints and pose the following guiding questions.

Treat this as a function where:

- Input is time (in hours)
- Output is distance (in miles)

Why would this function be continuous? Why is it important to recognize this is a continuous function?

Remember this is a calculus class, so we tend to look at rates. What rate applies to the scenario? What units would you use for this rate?

Have the partners find another set of partners so that groups of four are now working together. Let them work on this for a few minutes as you circulate from group to group.

On the first, quick pass, make sure that all students are setting up the function with time as the input in hours. If needed prompt students to set 2:04 pm to be t = 0 hours, and then have them calculate that 2:14 pm is t = ⅙ hours. Then prompt them to consider the rate which will be in miles/hour. [It is helpful for students write this out to see that is; some do not notice this because they are used to abbreviating mph.]

Once all students are directed as above, give students another hint.

Consider both the average and instantaneous rates of change for the two inputs that are given.

Make a quick second pass to prompting students, as needed, to come up with the facts that the average rate of change must be calculated. It is:

$$\frac{\Delta\, distance}{\Delta\, time} = \frac{(21-6)\ miles}{\left(\frac{1}{6}-0\right)\ hours} = 90\ miles/hour$$

While the two instantaneous rates of change can be gleaned from the given information in the scenario:

$$f'(0) = 60\ miles/hour$$

$$f'\left(\frac{1}{6}\right) = 63\ miles/hour$$

While circulating remind students that they will need to create a visual that represents the situation. Ask them what the information they should have come up with implies and ask them to try to show this on a xy-graph.

Optional Concluding Activity:

[Once this is done, you can finish the activity, but it is fun to have a what I call a *collab-etition* (a play on the words collaboration and competition). I let students know that I don't want to do too much grading; so, I will just grade one paper from the group to go forward. They are given a minute or two to decide. After a couple of minutes, I then say that I've changed my mind and that I really don't want to do too much grading. So, I have pairs of groups unite and determine which of their two papers will be graded. Depending on the number of groups, this might require involving three groups. After a couple of minutes, I then say that I've changed my mind and that I really don't want to do too much grading. So, the left half of the room will put the paper that goes forward for their side and the right half will pick the paper that goes forward for their side. Finally, I say that the two sides will go head-to-head to see which paper goes forward. There is a whole class vote. The only caveat is that the person who speaks for the paper must not have been in the group from where the paper came. Students seem to love this. They love getting to vote, and I have never had an instance where the whole class doesn't get a 100% for the activity. They also start to critically analyze the work, and I notice that work tends to improve for the entire class after this activity.]