The Limit of a function at a point

The graph of $f(x) = \frac{\sqrt[3]{x+3}-2}{x-5}$ has a hole. Your task is to determine the location of this hole using approximation techniques (no fancy limit computations allowed).

Preparation Instructions: Answer the following questions *individually* and bring your write-up to class.

A. Draw a graph of f using an entire sheet of paper. Your graph should be drawn at a scale that gives a good sense of the x,y-coordinates of the hole. The x and y scales should be chosen so that your graph nearly extends between two diagonally opposites corners of the page.

B. Identify what **unknown numerical value** you will need to approximate. Give it an appropriate shorthand name (that is, a variable name).

C. Describe what you will use for **approximations**. Write a description of your answer using algebraic notation (for example, function notation, variables, formulas, etc.)

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Class Instructions: Work with your group on the problem assigned to you. We encourage you to collaborate both in and out of class, but you must write up your responses *individually*.

1. Find an approximation to the height of the hole in your function (write out the approximation with several decimal places). Is this an **underestimate** or **overestimate**? Explain how you know. Find both an underestimate and an overestimate.

2. Redraw your graph at a good scale to clearly illustrate how you can approximate the height of the hole. Label the unknown height and the approximation.

3. Illustrate the **error** for your two approximations on your graph. Explain why you can't determine the numerical values of these errors. What is an algebraic representation for the error in your approximations?

4. Use your underestimate and overestimate to find a **bound on the error** for these two approximations. Explain your work. Illustrate this error bound on your graph.

5. List three fairly decent pairs of underestimates and overestimates (you can include the one you computed above). For each pair, give a bound for the error and use this to determine a range of possible values for the actual *y*-value of the hole in a table with headers as shown.

Underestimate	Overestimate	Error Bound	Range of Possible Values

6. Find an approximation with error smaller than 1×10^{-5} . Then describe as best as you can *all* of the *x*-values you could use to get approximations that would have an error smaller than this error bound.

7. For any pre-determined error bound, can you find an approximation with error smaller than that bound? Explain in detail how you know.