Systems of Linear Equations Exploration (Instructor Version with Solutions):

Part One (Tables): Seven-year old Noah had a collection of 20 coins. He saved only dimes and quarters (since they have bigger values than nickels and pennies). The total value of his coin collection was $3.95.

1) Consider some values that will allow you to satisfy the requirement of Noah having 20 coins and write them in the table below.

   Students will pick values that they choose to satisfy the 20 coin requirement. Sample values are provided below.

Dimes	Quarters	Total
10	10	20
5	15	20
15	5	20
1	19	20
20	0	20

a) Note: The total amount of coins is 20 for each combination. Is the monetary value of each of these options the same? Explain your reasoning.

The monetary value of each combination will change based on the number of dimes and number of quarters.

b) This requirement could be represented by the equation $d + q = 20$, where d represents the number of dimes and q represents the number of quarters.

i) What type of equation is given here?

Linear

ii) If the number of dimes is reduced by one, what effect does that have on the number of quarters?

The quarters must be increased by one.

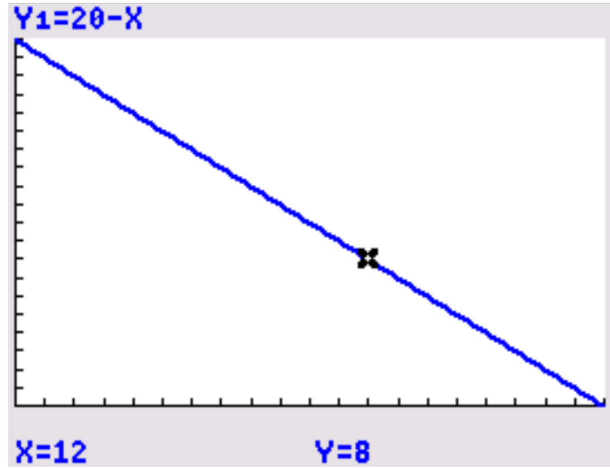
iii) If the number of quarters is increased by four, what effect does that have on the number of dimes?

The dimes must be decreased by four.

c) A student said 12 dimes and 8 quarters is a solution to this linear equation. Describe what the student means by solution.

The amount of 12 dimes and 8 quarters is a solution because $12 + 8 = 20$, which is the total number of coins.

d) Consider the graph of the linear equation $d + q = 20$ provided to the right. A point is labeled on the graph. How does this point relate to the answer to part c?



The point is representing 12 dimes and 8 quarters, which is the solution described in point c.

e) Notice that the equation was rewritten as $q = 20 - d$ in the provided graph.

i) What is the y-intercept of this graph? What does it represent in terms of this scenario?

The y-intercept or initial value is 20. This means that if there are 0 dimes selected, then all 20 coins must be quarters.

ii) What is the slope of this graph? What does it represent in terms of the scenario?

The slope of the graph is -1, which means with each added dime, the number of quarters decreases by 1. In other words, with each additional x-value (dimes), the y-value (quarters) decrease by 1.

2) Consider some values that will allow you to satisfy the requirement of Noah having a value of $3.95 in coins and write them in the table below.

Dimes	Quarters	Total
37	1	$3.95
32	3	$3.95
12	11	$3.95
7	13	$3.95
2	15	$3.95

a) Note: The total amount of monetary value for each combination is $3.95. Is the number of coins of each of these options the same? Explain your reasoning.

The number of coins will change based on the number of dimes and number of quarters needed because dimes and quarters are worth different monetary values.

b) This requirement could be represented by the equation $0.10d + 0.25q = 3.95$, where d represents the number of dimes and q represents the number of quarters.

i) What type of equation is given here?

Linear

ii) If the number of dimes is reduced by five, what effect does that have on the number of quarters?

The quarters must be increased by two.

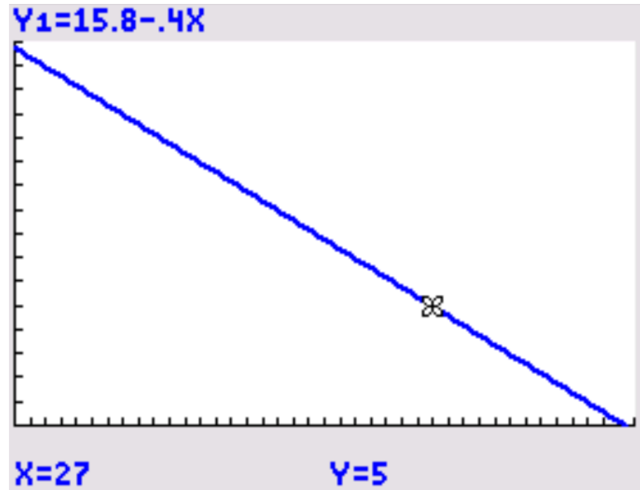
iii) If the number of quarters is increased by four, what effect does that have on the number of dimes?

The dimes must be decreased by ten.

c) A student said 27 dimes and 5 quarters is a solution to this linear equation. Describe what the student means by solution.

The amount of 27 dimes and 5 quarters is a solution because $0.10(27) + 0.25(5) = 3.95$, which is the total monetary value of the coins.

d) Consider the graph of the linear equation $0.10d + 0.25q = 3.95$ provided to the right. A point is labeled on the graph. How does this point relate to the answer to part c?



The point is representing 27 dimes and 5 quarters, which is the solution described in point c.

e) Notice that the equation was rewritten as $q = 15.8 - 0.4d$ in the provided graph.

i) What is the y-intercept of this graph? What does it represent in terms of this scenario?

The y-intercept or initial value is 15.8. This means that if there are 0 dimes selected, then 15.8 quarters would be needed to get \$3.95. This should be noted as impractical since quarters and dimes must be limited to whole numbers.

ii) What is the slope of this graph? What does it represent in terms of the scenario?

The slope of the graph is -0.4, which means with each added dime, the number of quarters decreases by 0.4. In other words, with each additional x-value (dimes), the y-value (quarters) decrease by 0.4. Again, this is impractical. However, this could be expanded to say that -0.4 is equivalent to $-\frac{2}{5}$, which could be interpreted (with whole numbers) as increasing the x-value (dimes) by five will result in reducing the y-value (quarters) by two. In terms of value, the dimes must be exchanged by fives with the quarter since \$0.50 is the smallest equivalent monetary value for dimes and quarters. This also relates to what was asked in part b.

3) Refer back to your tables. As you explored, did you choose the same values (or solutions) for each table? Why or why not?

Answers will vary. However, one observation students might notice is that they must have an odd number of quarters in order to satisfy the requirement of \$3.95. This value requirement is also more "strict" than the number of items (meaning there are fewer options).

4) Continue to explore until you find the number of dimes and quarters that would satisfy both requirements at the same time.

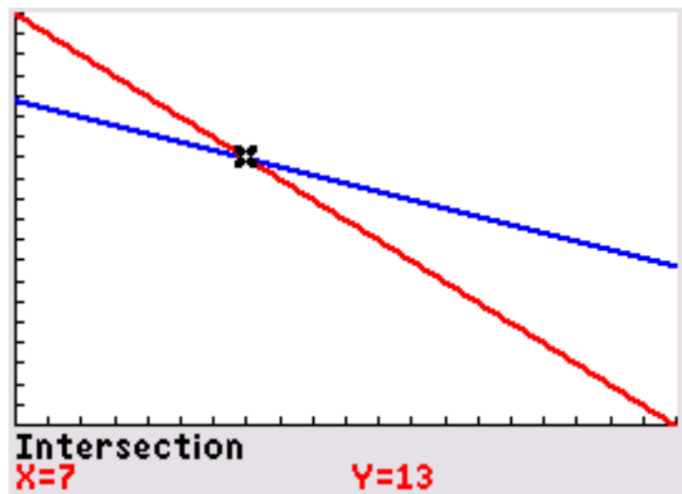
Dimes	Quarters	Total		Dimes	Quarters	Total
7	13	20		7	13	$3.95
		20				$3.95
		20				$3.95
		20				$3.95
		20				$3.95

You found the solution to the problem of determining the number of dimes and quarters given in the bag (with the specific requirements).

7 dimes and 13 quarters

This answer is called the solution to the linear system because it satisfies both equations in the system.

5) The graph to the right shows both previous graphs together on the same screen with a point labeled.



How would you describe this point?

The point provided shows the intersection of the two lines. It is the solution to the system because it is the one point that is on both graphs.

What does it mean in terms of the scenario?

This point is the solution representing 7 dimes and 13 quarters that satisfies the scenario of Noah having 20 coins with a value of $3.95.

Part Two (Equations): Olivia is creating a paved patio with space for 120 brick pavers that will contain both large square brick pavers and small rectangular brick pavers. Large square brick pavers cost $4.24 each and small rectangular brick pavers cost $2.59 each. Her budget is $390.00 for the entire patio. How many square brick pavers and how many rectangular brick pavers can she buy and stay with in her budget?

Note: In this problem, there are still two constraints on Olivia's patio. She must have 120 brick pavers and she must spend $390.00. However, in part two, the goal is to work with equations rather than tables.

1) Given s represents the number of square brick pavers and r represents the number of rectangular brick pavers, write an equation that shows the requirement of having a total of 120 brick pavers in the patio.

$$s + r = 120$$

2) Given s represents the number of square brick pavers and r represents the number of rectangular brick pavers write an equation that shows the requirement of having a budget of $390.

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$$4.24s + 2.59r = 390$$

3) What type of equations did you write? What would the graph of each equation look like?

These equations are both linear equations (in standard form).

The graph of each equation would be a line.

4) To make a visual of your answer, you should graph each equation.

a) First, solve each equation for s.

$$s = 120 - r$$

$$s = \frac{390 - 2.59r}{4.24}$$

b) Next, describe an appropriate window for this graph. Explain how you chose the values.

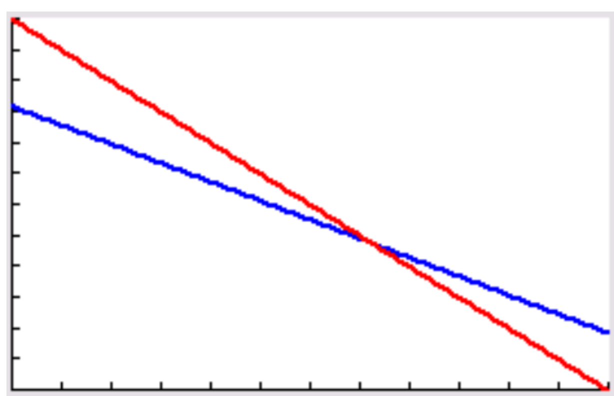
Answers may vary. But, one easy suggestion for the window is xmin and ymin both being 0 and xmax and ymax both being 120. This one uses the maximum amount of brick pavers as the maximum for each (maybe the patio has 120 square bricks and 0 rectangular bricks, for example). The minimum is 0 since the scenario would not allow for a negative amount of brick pavers .

c) Type the equations in $Y1$ and $Y2$. Then graph using your chosen window. Sketch your graph and label axes.

$$Y1 = 120 - x$$

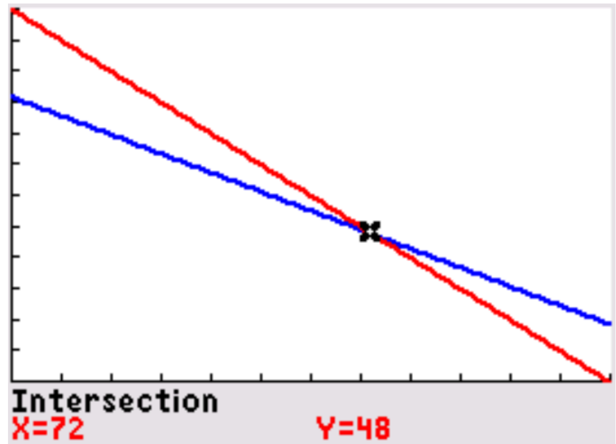
$$Y2 = \frac{390 - 2.59x}{4.24}$$

Note: Y is representing square brick pavers in this problem and x is representing rectangular brick pavers.



d) What do you notice about the graphs?

The graph on the calculator shows two intersecting lines.



e) Identify the intersection point from the graph. How do you think that point relates to the scenario and the number of bricks?

The intersection point (48, 72) is the point where the two lines meet. This point gives the values that satisfy both equations/requirements of the problem. This means Olivia should have 48 square brick pavers and 72 rectangular brick pavers in her patio.

Note to instructors: Question 5 is included to make clear to students that either variable can be isolated and the same solution (in terms of the scenario) can be found (with the ordered pair coordinates reversed). Some instructors may find it unnecessary and can proceed to part three.

5) Now, let's try that again. To make a visual of your answer, you should graph each equation.

a) First, solve each equation for r.

$r = 120 - s$

$r = \frac{390 - 4.24s}{2.59}$

b) Next, describe an appropriate window for this graph. Explain how you chose the values.

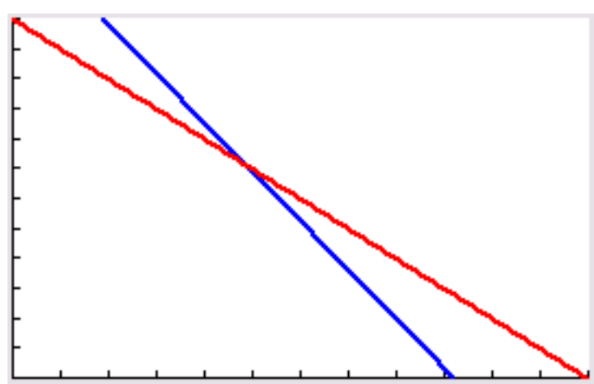
Answers may vary. But, one easy suggestion for the window is xmin and ymin both being 0 and xmax and ymax both being 12. This one uses the maximum amount of brick pavers as the maximum for each (maybe the patio has 12 square bricks and 0 rectangular bricks, for example). The minimum is 0 since the scenario would not allow for a negative amount of brick pavers .

c) Type the equations in $Y1$ and $Y2$. Then graph using your chosen window. Sketch your graph and label axes.

$Y1 = 120 - x$

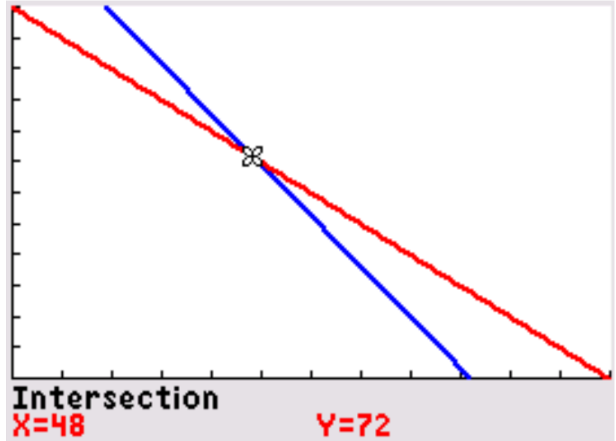
$Y2 = \frac{390 - 4.24x}{2.59}$

Note: Y is representing rectangular brick pavers in this problem and x is representing square brick pavers.



d) What do you notice about the graphs?

The graph on the calculator shows two intersecting lines.



e) Does any particular point match your solution found in 4e? Identify the point and describe what it means in terms of the scenario. How does this answer compare with part 4e?

In part 4e, the intersection point was (48,72), but now the intersection is (72, 48). This represents 72 square brick pavers and 48 rectangular brick pavers, but in the opposite order (due to the opposite variable being isolated). So, each way of graphing found the same solution to Olivia's patio problem.

Part Three (Algebra): For a brunch, Liam is bringing mini carrot cake muffins and hard-boiled eggs. Liam would like to bring three times as many carrot cake muffins as hard-boiled eggs. Each mini muffin cost \$0.75 and each egg cost \$0.22. Liam paid a total cost of \$17.29 for his brunch items. How many muffins and eggs did Liam bring to the brunch?

Note: In addition to using tables or graphing systems of equations, there are several algebraic approaches to solving linear systems of equations. One option that works well with the previous steps is substitution.

1) Describe the two requirements that Liam is following for his brunch items.

Students should notice that one requirement is similar to the previous one with having \$17.29 as a budget. The other requirement comes with the items in that Liam wants three times as many muffins as he does eggs.

2) Write two equations with each one representing one of the requirements described in part 1. Use m to represent carrot cake muffins and h to represent hard-boiled eggs.

$m = 3h$

$0.75m + 0.22h = 17.29$

3) Solve the equations for m. Why is it beneficial to solve both equations for m?

$m = 3h$

$m = \frac{17.29 - 0.22h}{0.75}$

Solving for m is easier in this scenario because m is already isolated in the first equation.

4) Set the equations equal to each other (using substitution) to solve for h.

$m = 3h$ $\qquad\qquad$ $m = \frac{17.29-0.22h}{0.75}$

$3h = \frac{17.29-0.22h}{0.75}$

$2.25h = 17.29 - 0.22h$

$2.47h = 17.29$

$h = 7$

So, Liam should bring 7 hard-boiled eggs.

5) To find m, plug in 7 for h in either equation from part 3.

$m = 3(7) = 21$ $\qquad$ OR $\qquad$ $m = \frac{17.29-0.22(7)}{0.75} = 21$

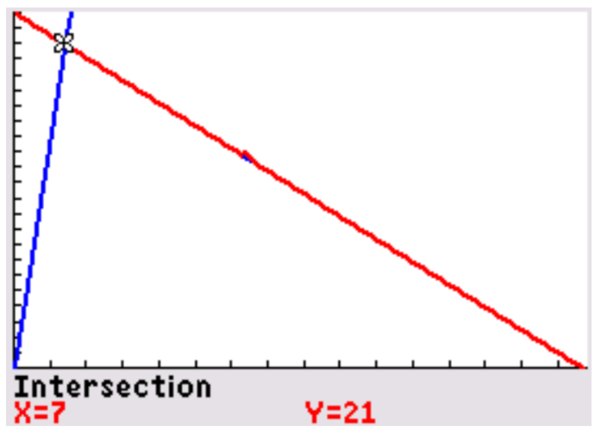
So, Liam should bring 21 mini carrot cake muffins.

6) Use the equations from part 3 to check your solution by graphing. Describe an appropriate window.

$m = 3h$

$m = \frac{17.29-0.22h}{0.75}$

One option for the window is to use xmin = 0 and xmax = 79 and ymin = 0 and ymax = 23. These max values are set by finding how many eggs or muffins Liam could buy if he only bought one item (all egg or all muffins). It isn't the best window (see graph to the right) since it has quite a lot of empty space, but it would be appropriate for this scenario.



Part Four (Independent): Solve each system of equations problem by writing two equations that represent the requirements of each scenario. Then solve using a method of your choice.

This section is meant for independent practice for the students and can be done individually or in small groups. They may choose to use tables, solve by graphing, or solve algebraically. As time permits, discussion about the advantages and disadvantages of each method could be done.

a) Elijah is restocking his household with flashlights and batteries. He finds an online deal for flashlights for \$4.99. Each flashlight needs three (3) AAA batteries, which he found for \$0.36 each. His pre-tax total is \$24.28. How many flashlights and batteries did Elijah purchase?

$4.99F + 0.36B = 24.28$
$B = 3F$

4 flashlights, 12 batteries

b) Amelia teaches a geometry class and is in need of protractors and rulers. She finds protractors for $0.35 and rulers for $0.74 each from a teacher resource store. She purchases a total of 59 protractors and rulers. How many protractors and rulers did Amelia purchase?

$0.35P + 0.74R = 35.47$
$P + R = 59$

21 protractors and 38 rulers