**The Quadratic River Problem**

**Problem Instructor Guiding Questions:**

## MIPPED Activity (Instructor Version):

**Part One:** 1. How is area and perimeter calculated for rectangles? When might it be helpful to maximize or minimize area in real-life? Perimeter?

Students should recall that the area of a rectangle is found by multiplying the base times the height or the width times the length of a rectangle. Students should recall that the perimeter of a rectangle is found by adding up all of the side lengths. Answers will vary on real-life applications, but may include things such as farming, gardening, fencing, playgrounds, etc.

Find the area and perimeter rectangles with the given widths and lengths to fill out the table below.

| Width | Length | Perimeter | Area |
|---|---|---|---|
| 100 ft | 200 ft | 600 ft | $20000\ ft^2$ |
| 50 ft | 250 ft | 600 ft | $12500\ ft^2$ |
| 125 ft | 175 ft | 600 ft | $21875\ ft^2$ |

What do you notice about the perimeters? Areas?

Students should access prior knowledge about perimeter being the sum of all side measures and the area being length times width. They should notice that each given width and length has the same perimeter, but quite different areas. Students should include units for perimeter and area to reinforce quantitative reasoning.

2. Let's explore more with this online geogebra tool. Use the perimeter of _________ units.

The instructor must pick a given perimeter and tell the students they need that same perimeter throughout the table/tool exploration. Then have students find widths, lengths, and areas that align with the given perimeter. In this sample table, we picked 60 for the perimeter and noticed that three different rectangles give us three different areas. We did pick one where the rectangle happened to be a square and that looks like a maximum area.

| Width | Length | Perimeter | Area |
|---|---|---|---|
| 10 units | 20 units | 60 units | 200 units squared |
| 15 units | 15 units | 60 units | 225 units squared |
| 5 units | 25 units | 60 units | 125 units squared |

3. What do you notice from your explorations?

Students should notice similar ideas to the first table where the perimeter is the same for three different rectangles with different areas. From the sample table, the square has the biggest area.

4. Continue to work with the geogebra tool. Given the perimeter of _____ units 60 units (use the same perimeter as before), what width and length maximizes the area? What width and length minimizes the area? A table is provided for student use, if needed.

Students may pick various widths and lengths to explore, but a sample table is provided below.

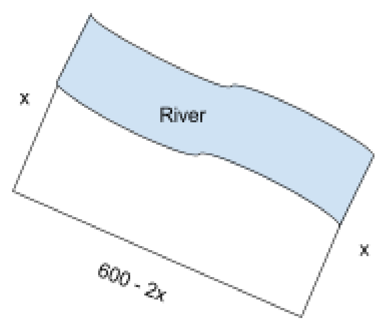
| Width | Length | Perimeter | Area |
|---|---|---|---|
| 3 units | 27 units | 60 units | 81 square units |
| 2 units | 28 units | 60 units | 56 square units |
| 1 units | 29 units | 60 units | 29 square units |
| 14 units | 16 units | 60 units | 224 square units |
| 13 units | 17 units | 60 units | 221 square units |
| 12 units | 18 units | 60 units | 216 square units |
| 15 units | 15 units | 60 units | 225 square units |

Students should notice that "skinny" rectangles result in the least amount of area and more "square" rectangles (where the width and length have closer values) result in the most amount of area.

5. Suppose you have 600 feet of fencing to enclose a rectangular plot that borders on a river. In this rectangular plot, you are building a garden.

a. Draw a picture to match the scenario. Pick the two congruent sides to be $x$ and label the other side(s) accordingly. Describe what $x$ is represented in your picture.

Students should know that $x$ is the side length for one side of the rectangle. They should recognize that the units are in feet. The other side should be labeled as $600 - 2x$. A sample picture is provided.

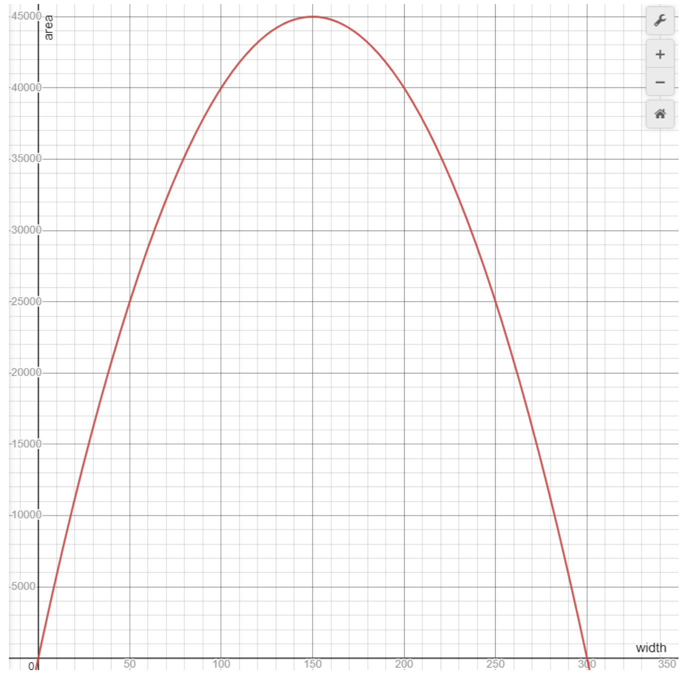



b. Using what you know about calculating the area of a rectangle along with the picture from part a, write an equation for Area, $A(x)$ of the rectangular plot.

Students should find the formula to be $A(x) = x(600 - 2x) = -2x^2 + 600x$. They should note that units for area are square feet. They should recognize this as a quadratic equation where the graph is a parabola that opens down. Students may choose to have the form in standard form (as written above) in order to more easily identify the coefficients of a, b, and c in the quadratic equation.

c. Graph $A(x)$ on your graphing calculator. Sketch a graph labeling your axes appropriately, including units.

Provided is a sample graph from Desmos.




d. Using your graphing calculator to complete the following table. Make sure to include appropriate units. Remember that the graph in part c is a graph of the area function $A(x)$ for the rectangular plot.

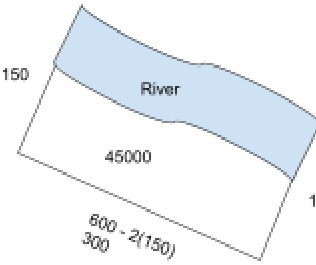
| Width x | Length | Perimeter | Area |
|---|---|---|---|
| 20 ft | 560 ft | 600 ft | $11200\ ft^2$ |
| 50 ft | 500 ft | 600 ft | $25000\ ft^2$ |
| 130 ft | 340 ft | 600 ft | $44200\ ft^2$ |
| 150 ft | 300 ft | 600 ft | $45000\ ft^2$ |

e. What do you notice from the graph and the table? Include information about perimeter, width, and length. Are there any area values that have multiple options for width and length? Explain.

We hope that students say perimeters are the same (yes, it seems obvious, but it may not be something students notice). They should also notice that each area happens twice on the graph (once on the increasing side and once on the decreasing side). It would be beneficial to notice that there is a maximum area at the vertex, but if not, they will get that in the next couple questions.

f. Since we are fencing in a garden, we want to maximize the area. Find the width, length, and area associated with maximizing area using the graph. Describe the point that corresponds to these values. Redraw your original picture with labels for the found width, length, and area measurements.

Based on the graph/table, the maximum area is 45000 square feet with a width of 150 ft and a length of 300 ft. This is shown by comparing values in the table, but it is also the vertex of the graph.




g. If you computed $A(150)$, what value would you expect to get? (Do not compute the value).

Without computing A(150), students should expect the value to be 45000 square feet because A(150) represents the area when the width is 150 ft, which is described in part f.

h. What point does the ordered pair $(\frac{-b}{2a}, A(\frac{-b}{2a}))$ represent on the graph?

The ordered pair represents the vertex on the graph. It is also the width and area when the area is maximized.

i. Find the vertex of the graph of $A(x)$ (using $x = \frac{-b}{2a}$ to find the x-coordinate). Compare this with your answer in part f.

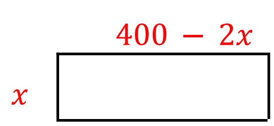
$x =- \frac{b}{2a} =- \frac{600}{2(-2)} = \frac{-600}{-4} = 150$. This means the width of the rectangle should be 150 ft.
$y = f(-\frac{b}{2a}) = f(150) =- 2(150)^2 + 600(150) = 45000$. This means that the area of the rectangle will be 45000 $ft^2$.

This matches!

Once this activity has been completed, it would seem beneficial to the students to either further explore the idea via another similar activity where students must recognize that the vertex provided information about the scenario's minimum or maximum value.

**Part Two:** Suppose you have 400 feet of fencing to enclose a rectangular plot. In this rectangular plot, you are building a playground.

a. Draw a picture to match the scenario. Pick the two congruent sides to be $x$ and label the other sides accordingly.



b. Using what you know about calculating the area of a rectangle along with the picture from part a, write an equation for Area, $A(x)$.

Students should find the formula to be $A(x) = x(400 - 2x) = -2x^2 + 400x$

c. Complete the following table. Make sure to include appropriate units.

| Width x | Length | Perimeter | Area |
|---|---|---|---|
| 30 ft | 170 ft | 400 ft | 5100 $ft^2$ |
| 50 ft | 150 ft | 400 ft | 7500 $ft^2$ |
| 80 ft | 120 ft | 400 ft | 9600 $ft^2$ |
| 100 ft | 100 ft | 400 ft | 10000 $ft^2$ |

d. Since we are fencing in a playground, we want to maximize the area. Find the width, length, and area associated with maximizing area using $(\frac{-b}{2a}, A(\frac{-b}{2a}))$.

$x =- \frac{b}{2a} =- \frac{400}{2(-2)} = \frac{-400}{-4} = 100$. This means the width of the rectangle should be 100 ft.

$y = f(- \frac{b}{2a}) = f(100) =- 2(100)^2 + 400(100) = 10,000$. This means that the area of the rectangle will be 10,000 $ft^2$.

e. Based on the information from the table and your answers from part d, what shape would give us the maximum area?

Students should be able to answer this question based on parts c and d, since the area is maximum when width is equal to the length (a square!).