Absolute Value Discovery – Solutions

Mark off a length of tape (masking or painters) on the floor around 5 feet long. Do this for each group.

One student from each group should be chosen to use their foot as a unit of measurement. Each group should go to the tape, marked off for their group on the floor, and measure the distance by stepping heel to toe. Each step is one unit. Estimate the distance by how many footsteps were taken.
Distance of the tape

Answers will vary depending on the length of tape and the person's foot size

Now the same student will measure the same strip of tape by stepping heel to toe backwards.
Distance of the tape

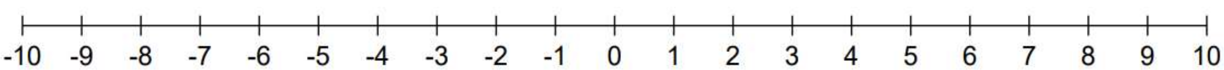
Answers will vary depending on the length of tape and the person's foot size

Does the distance change based on the direction the steps were taken?

Explain what was discovered by this activity.

The students should discover that distance is not negative. You can also guide them into understanding that a unit of measure can be anything, for instance a pen length, a person's foot, a sheet of paper or any other consistent tool they choose to use for measurement.

Defining Absolute Value



Use the number line to determine each of the following:

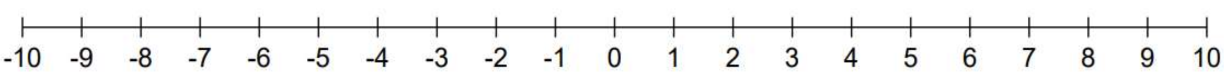
1. the distance from 3 to 7 distance is 4

2. the distance from 7 to 3 distance is 4

Does the distance change based on the direction you move?

No, distance is the same, use this as an opportunity to dispel misconceptions about the distance from 7 to 3 being –4.

Explain: The students should discover once more that the direction of movement does not change the distance to a negative. This would be a good time to have students share their explanations.



Use the number line to determine each of the following:

3. the distance from –4 to 6 distance is 10

4. the distance from 3 to –2 distance is 5

Explain what you have learned about distances from these examples.

Distances are always positive. Students should also realize with this portion of the activity that the direction they move does not matter. This is a good place to use a comparison of driving 50 miles east or 50 miles west to show that the direction does not matter. They are driving 50 miles regardless.

Starting at 0 and moving the same distance you determined from questions 1 & 2,

where could you end up on the number line? Final location is 4 or – 4

Using this discovery write a definition in your own words for absolute value.

Absolute value is the distance a number is from zero, therefore, it will equal a positive number

Mathematical symbols for absolute value

$|x| = 4$ means the absolute value of a number x is 4

Using your definition from above, write an explanation of the above statement.

Both 4 and – 4 are a distance of 4 places from zero. When possible, always stress the idea of distance to students.

Solving absolute value equations

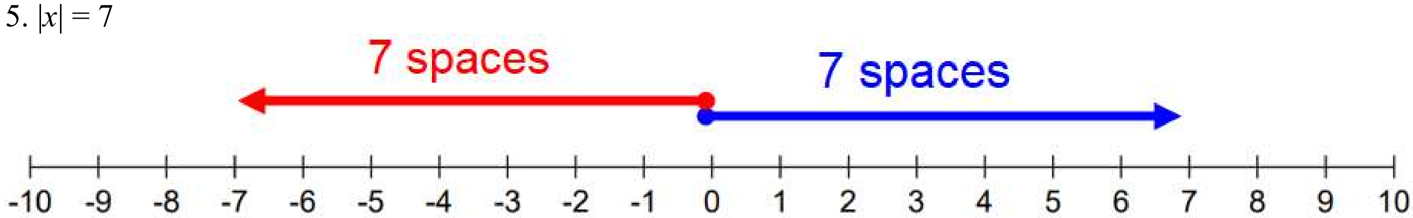
If $|x| = 4$ from previous discoveries, we know we would be moving 4 spaces from 0.

This means we could end up at 4 or – 4

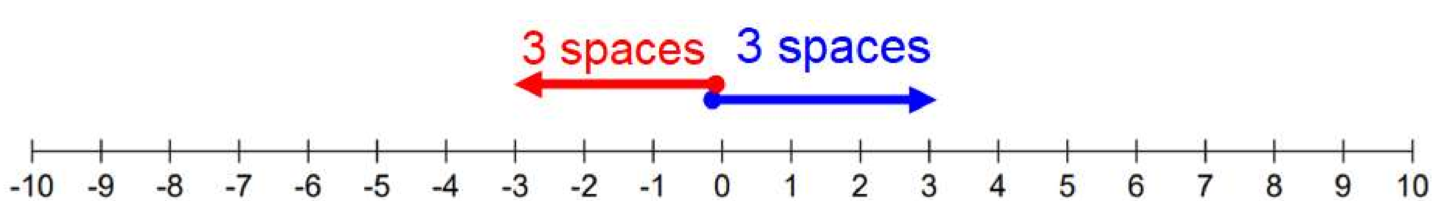
Therefore, if $|x| = 4$ $x = 4$ or $x = -4$ Either one is 4 spaces from 0.

Using this rule or your definition what would be the solutions for x in the following equations and express each problem on a number line?

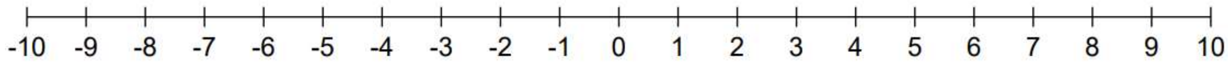
5. $|x| = 7$



6. $|x| = 3$



7. $|x| = -4$ Not possible to move a negative distance on a number line.



What did you discover about moving a negative distance?

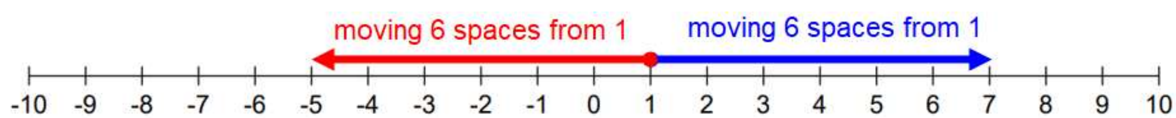
You cannot move a negative distance, regardless of whether you move left or right the distance is positive. Asking students about measuring with a tape measure or ruler and pointing out that there are no negative numbers on these helps them realize this idea.

How does that affect your decisions when solving an absolute value equation equal to a negative number?

There would be no solution, since you cannot measure a negative distance.

Now you will discover how to apply this concept if the value inside the bars was more than just an x.

1. Choose a number on the number line.
2. Choose a distance to move in both directions from your chosen number.
3. Draw segments to each of those numbers.



What were the two solutions?

This has multiple possibilities depending on where the student chooses to start and how far they move. Discuss the fact that the two answers are not the same number, and are not exact opposites.

You learned earlier that an absolute value cannot equal a negative number because it is a distance. But keep in mind you are moving in either a positive or negative direction to reach your two solutions.

Write an absolute value equation that would match the segment you just drew on the number line. Keep in mind that absolute value is equal to a distance. Think about satisfying the |positive| and |negative| values.

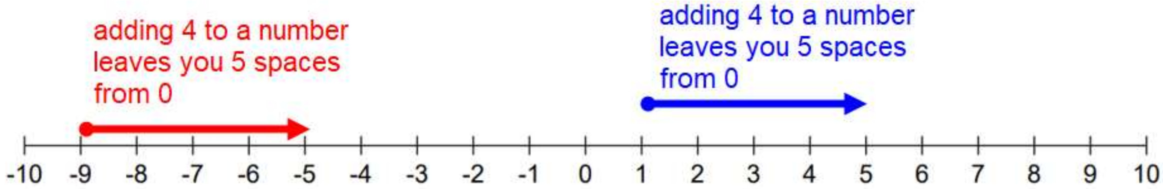
Let the students contemplate what the equation would be. This will take some thought and it is ok if they don't get it correct. Use this as discussion time on if it is $|x + a|$ or $|x - a|$ inside the absolute value bars.

8. If $|x + 4| = 5$ explain in words how you would solve this equation?

If I move $x + 4$ spaces it will equal a distance of 5 from 0 taking me to 5 or -5. Let the value $x + 4 = 5$ and $x + 4 = -5$ the solve for x by subtracting 4 from both sides of each equation.

Be sure that the students have fully developed the understanding that they need two equations, one equal to -5 and one equal to 5.

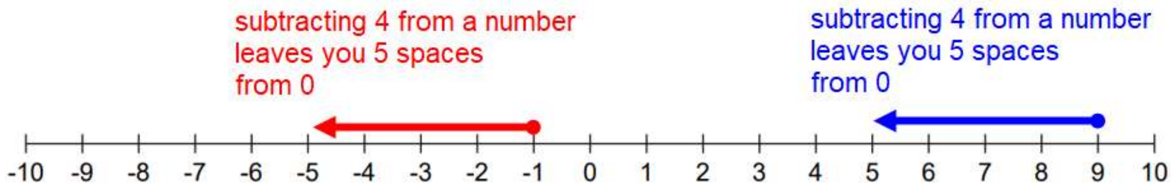
What would this look like on a number line?



9. If |x – 4| = 5 explain in words how you would solve this equation?

Let the value x – 4 = 5 and x – 4 = – 5 the solve for x by adding 4 from both sides of each equation.

What would this look like on a number line?



10. Using the same idea, create two absolute value equations that equal 6.

Multiple answers are possible. Let the students play around with creating various equations.

Some examples $|x - 2| = 6$ $|x + 4| = 6$ $|5 - x| = 6$

Show your work for each of the following:

11. $|x + 3| = 9$

12. $|x - 2| = 11$

First split into 2 equations

$x + 3 = 9$ $x + 3 = -9$ (– 3 on both sides) $x - 2 = 11$ $x - 2 = -11$ (+2 on both sides)

$x = 6$ $x = -12$ $x = 13$ $x = -9$

Ensure that students do not only solve the first and think the second solution is just the opposite value.

Write an explanation of your thinking process for solving one of the equations above.

Check to ensure they have created 2 equations in their explanation. A common mistake is forgetting that they will have 2 solutions requiring 2 separate equations. This could create a good discussion in small groups to let students dispel their own misconceptions.

More advanced absolute value equations.

13. $4|x + 2| - 2 = 14$

- When solving an equation in the form $3x - 2 = 13$, the first step is to begin to isolate the variable by adding or subtracting the constant to both sides.

What do you think the first step in solving this equation should be?

Isolate the absolute value by first adding 2 to both sides of the equation. Then dividing by 4 on both sides. Ensure that students do not treat the absolute value bars like parenthesis and try to distribute the 4 inside the absolute value.

Show the step here.

$4|x + 2| - 2 = 14$

$$
\begin{array}{l}
4|x + 2| - 2 = 14 \\
\qquad\quad +2 \quad\; +2 \\
4\,|\,x + 2\,| = 16
\end{array}
$$

- The equation from above is now $3x = 15$. The next step is to divide by the coefficient (number in front) of the variable.

What do you think the second step in solving this equation should be?

Then divide by 4 (the number in front of the absolute value) on both sides to get just the absolute value alone on one side.
Ensure that students do not treat the absolute value bars like parenthesis and try to distribute the 4 inside the absolute value.

Show the step here.

$4|x + 2| = 16$

$$
\begin{array}{l}
4\,|\,x + 2\,| = 16 \\
\div 4 \qquad\quad \div 4 \\
|\,x + 2\,| = 4
\end{array}
$$

(divide both sides by 4 to isolate the absolute value)

Now the equation should be similar to problems 11 and 12. Finish solving the equation here.

$|x + 2| = 4$

$|\,x + 2\,| = 4$ (split into 2 equations)

$x + 2 = 4$	$x + 2 = -4$
$-2 \quad -2$	$-2 \quad -2$
$x = 2$	**$x = -6$**

(subtract 2 from both sides of each equation)

Repeat the process to solve the following equations.

13. $2|x - 7| + 6 = 20$

$2|x - 7| + 6 = 20$

$\quad\quad -6 \quad -6$

$2|x - 7| = 14$

$\div 2 \quad \div 2$

$|x - 7| = 7$

split into 2 equations

$x - 7 = 7$ $\quad\quad$ $x - 7 = -7$

$+7 \quad +7$ $\quad\quad$ $+7 \quad +7$

$x = 14$ $\quad\quad$ **$x = 0$**

14. $-3|2x - 5| + 3 = -21$

$-3|2x - 5| + 3 = -21$

$\quad\quad - 3 \quad - 3$

$-3\,|2x - 5| = -24$

$\div(-3) \quad \div(-3)$

$|2x - 5| = 8$

$2x - 5 = 8$ $\quad\quad$ $2x - 5 = -8$

$+5 \quad +5$ $\quad\quad$ $+5 \quad +5$

$2x = 13$ $\quad\quad$ $2x = -3$

$\div 2 \quad \div 2$ $\quad\quad$ $\div 2 \quad \div 2$

$x = 13/2$ $\quad\quad$ **$x = -3/2$**