Problem Name/Description: Linear Regression & Systems with Knots

In this problem, students will reinforce the steps of finding a linear regression model, describe the meaning of slope and y-intercept of the models, and find the solution to a system of equations, all while tying knots in a hands-on approach.

Rationale for selecting/designing this problem/task sequence:

- This problem involves many key ideas from Linear Functions in Functions & Modeling including linear functions, linear regression, and systems of linear equations. This problem physically has the students model what is happening with the functions and system so that they connect key ideas (from pre-requisite knowledge) to hands-on learning.

Prerequisite Knowledge:

- Students can tie knots.
- Students can measure to the nearest centimeter.
- Students can interpret slope (as steepness or growth rates).
- Students can interpret y-intercepts (as starting points where the x-value is 0).
- Students can create lines of best fit on the graphing calculator.

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Students can create scatter plots on the graphing calculator. (SLO 5)
- Students can explain the meaning of the intersection point of a system of equations and its application to the scenario. (SLO 1, 5)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Linear functions

Targeted understanding of the key idea/topic:

- For students to recognize the slope of a linear regression as an estimation of successive, equal changes in the input quantity corresponding to consistent changes in the output quantity
- For students to identify the y-intercept as the initial value
- For students to understand the intersection of two lines as giving the conditions when the output values of the two functions are equal

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
  - Transitional understanding vs targeted understanding

Students can tie knots.	Students can correctly record data. Students can measure to the nearest centimeter.

↓ *Students collect data about tying knots in two ropes of different thickness.*

Students can make scatter plots.

↓*Students make scatter plots and discuss the data.*

Students see the data as approximately linear.	Students can generate a linear regression.

↓*Students use the calculator to find linear regression equations for both ropes.*

Students interpret the slope and y-intercept of each equation.

↓*Interpret the slope and y-intercept as the rate of decrease in length of rope and the starting length of the rope.*

Students can graph scatter plots and regression models.	Students can solve equations with crossing graphs.

↓*Students find the point of intersection (graphically).*
↓*Students interpret the meaning of the intersection point of a system of equations.*

Students can untie knots.

↓*Check intersection point by untying knots on ropes.*

Students confirm the relationship between intersection point and the number of knots that give equal length ropes.

↓*What does it mean for the intersection to occur on the y-axis?*

Y-axis -> input is 0 -> 0 knots -> initial length of ropes -> same length

↓*What would happen if the thicker rope was the shorter rope?*

Thicker rope -> more negative slope -> thinner rope unable to "catch up"-> intersection point occurs with negative amount of knots

Students can solve linear systems by hand.

↓*Students find the point of intersection (algebraically).*

See that algebraic intersection and graphical intersection are same.

Problem Instructor Guiding Questions:

Given these two ropes (of unequal lengths and thickness), collect and analyze data to discuss slope, intercepts, and intersection points of the graph.

*This works best when the thicker rope is longer and the thinner rope is shorter. However, other options are presented in the class discussion.

Materials: 2 unequal lengths of different size rope, tape measure, graph paper, graphing calculator

Instructions: Start with one rope. Measure its length. Then tie one knot in the rope and measure its new length. Continue this process, measuring the length of the rope after each knot until you can no longer tie a knot easily. Repeat this procedure with the other rope. Record your data in this table.

*We suggest starting with a rope and demonstrating a basic knot along with having a short discussion on measuring, accuracy, etc.

* Have the same person tie the knot each time to encourage consistency.

Number of knots	Length (thinner rope) in cm	Length (thicker rope) in cm
0		
1		
2		
3		
4		
5		

a) Make a scatterplot of your data of the length of your rope versus the number of knots tied. Each rope should have a set of data points plotted. Label the axes and units appropriately.

*This could be done on a graphing calculator or by hand (but the graphing calculator is recommended). There could be a nice discussion about what the students see on the graph. They could look for trends (such as linear trends) and see both plots of data are decreasing. They might notice that one rope starts at a higher y-intercept than the other.

b) Use your graphing calculator to find the equations of the two lines (round to 3 decimal places). What do the slopes of the equations represent? What do the y-intercepts represent?

*Students should be interpreting slope as the decreasing in rope length with each additional knot tied.

c) Use your graphing calculator to find the intersection point of the two lines. Interpret the intersection point as the point where the number of knots and rope lengths match (or are equal) for both ropes.

*We recommend using crossing graphs to find the intersection point.

d) Can you physically prove your interpretation of the intersection point? Describe what would have to occur for the intersection point to be on the y-axis. Describe what would happen if the thicker rope was the shorter rope.

*This is a great point for discussion for the students. You can even pose the question and then give more time for group discussion (rather than class discussion) if students cannot imagine what would occur. You might also have a pair of ropes handy for each special case.

e) Solve the system algebraically (to find their intersection point) and describe how your answers compare to the point you got in question c. Is it the same or different? Why?

*Students should know that the answers should be the same. They should acknowledge that the graph, the knot tying, and the algebra are all three different ways to arrive at the same solution.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- Students are actively engaged throughout the entire problem. Students are physically tying knots, recording data, making predictions, and interpreting the results.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The student scenario has been shortened and made more open-ended to allow for more student discussion. Students discuss key ideas of slope, y-intercept and intersection points. Students evaluate what these key ideas mean when the intersection point is "special" (see part c in student scenario).

Optional extensions of the problem.

- This activity is already fairly active.
- Note this activity is intended for Functions & Modeling and it is expected that instructors assign this activity before students have learned about non-linear models. If assigned after learning about non-linear models, then the instructor could remove the instructions for the model being linear, and have students decide which model is most appropriate.

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- This problem is easily connected to the students since they can all tie knots and physically see the connection between linear equations and the knot tying process. After this problem, students should work more on systems of equations. With each problem, the intersection point being recalled as the point of equivalent x and y values should be emphasized.

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- Regression is a prerequisite topic that previously was an afterthought, but now has become the primary strategy for finding the linear equations. This helps students in this context, which has an input variable of a number of knots, as opposed to time.

Optional extensions of the problem.

- This problem could be extended by adding in a third rope. Students could generalize to the third rope and make predictions about what would happen based on its initial length and thickness.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- This problem allows students to jump into a problem at a low-entry level. This will help with math anxiety. Students develop classroom community by working together to collect the data. Students integrate technology by using the regression features on the graphing calculator. During class discussion, students can discuss what happened with the knots and develop critical thinking skills by envisioning various scenarios.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- Problem was re-written to be more open with the idea that students would work together in groups and more discussion would occur (more student-centered versus teacher-led). This helps build a sense of belonging in a mathematical community.

Optional extensions of the problem.

- N/A