Problem Name/Description: Linear Regression with Enrollment

In this problem, students find linear regression models for data relating to college enrollment. They interpret the slope and y-intercept and discuss alignment of the input values.

Rationale for selecting/designing this problem/task sequence:

- This problem focuses on what effect alignment (horizontal transformation) has on a linear graph, the slope, the intercept, etc. We use the term "alignment" to not only indicate a horizontal transformation, but a transformation that makes sense with the context of the problem. The most common example of alignment is with years. Instead of having an input value of 2021, you might align your inputs to be the number of years since 2020 and then have an input value of 1. Alignment is necessary in this course and in the following Business Calculus courses, so we would like students to have an understanding about alignment so that they can decide for themselves whether or not a data set should be aligned, and to what extent it should be aligned.

Prerequisite Knowledge:

- Generate linear regression using technology when utilizing data that is close to linear.
- Interpret slope in context.
- Extrapolate output values.
- Understand transformations - horizontal shifts

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Identify the similarities and differences when a function's inputs are aligned or not aligned, and make connections with the context as to why something is the same or different. (SLO 1, 2, 5)
    - Identify that the slope is the same, but the y-intercept is different.
    - Output predictions should be the same.
- Identify alignment as a horizontal shift. (SLO 4)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Linear Functions, Regression Lines

Targeted understanding of the key idea/topic:

- For students to recognize the slope of a linear regression as an estimation of successive, equal changes in the input quantity corresponding to consistent changes in the output quantity

- For students to identify that slope is invariant across multiple alignments of the input quantity, while the initial value is dependent on choice of alignment

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
    - Transitional understanding vs targeted understanding

Generate linear regression using technology.

↓ *Find linear regression (either aligned or unaligned).*

Interpret slope and y-intercept in context.	Extrapolate outputs.

↓ *Predict the enrollment in a certain future year.*
↓ *Find and interpret the slope and y-intercept.*
[get into groups]
↓ *Compare and contrast the two models.*

Same slope, different y-intercept, same extrapolation.	The slope is the same because the enrollment changes in the same way every year, regardless of if that year is aligned or not.

↓*Compare and contrast the graphs of the two models.*
↓*When we align the input data, what effect does that have on the scatterplot and on the regression function?*

Understand transformations - horizontal shifts	Identify alignment as a horizontal shift.

Problem Instructor Guiding Questions:

The table gives college enrollment in the United States for certain years.

Year	U.S. College Enrollment (million students)
2005	4.47
2006	4.58
2007	4.76
2008	5.13

(a-b) Some students will generate their model unaligned while some will align. Their predictions and slope should be the same, while their y-intercept will be different.

(c-d) Get the students together in groups to discuss these similarities and differences and why they occur. The students will likely give surface level comparisons and contrasts of slope and y-intercept being the same or not, so lead the discussion towards the invariance of slope and alignment being a horizontal shift, as needed. "When we align the input data, what effect does that have on the scatterplot and on the regression function?"

*Here is the understanding of slope that is targeted with this problem: The constant rate of change is the amount of increase in the output quantity when the input increases by 1.

After the task: Have students generalize the relationship between shifting and alignment by having your next regression activity be a context where the inputs are NOT a measure of time.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- Has students compare and contrast the two regression equations.
- Has students analyze the effects of alignment on a scatter plot.
- Students explain why the slope is invariant.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The student scenario has been shortened and made more open-ended to allow for more student discussion.
- Have students compare and contrast their scatter plots and regressions.They should look at key characteristics such as slope and y-intercept.
- Students select how to perform the linear regression (whether to align or not).

Optional extensions of the problem.

- Have students algebraically check that $g(x) = f(x + 2005)$

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- Has students identify the mathematical relationship between alignment and horizontal shifts through the use of regression.
- The instructor-led discussion asks students to generalize this relationship.

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- N/A

Optional extensions of the problem.

- Ask students to conjecture whether or not this relationship holds for all alignments and why.
- Ask specifically about contexts where the input variable is not time and what alignment would mean in those cases.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- Students use critical thinking skills to compare and contrast the two linear regression models. The resulting discussion allows students to see the reason why the inputs are typically aligned in this type of problem and why aligning does not invalidate the model.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- Problem was re-written to be more open with the idea that students would work together in groups and more discussion would occur (more student-centered versus teacher-led). This helps build a sense of belonging in a mathematical community.

Optional extensions of the problem.

- Nothing about this data is particularly vital to the structure of the overall problem. The instructor using this problem could find enrollment data for their University specifically and see if it is roughly linear. This can be a small way to contribute to the students' identity as members of the University and gauge interest in the problem.