Problem Name/Description: Creating and Analyzing Linear Functions

In this problem, students will be considering a given scenario involving the growth rate of a child. The students will need to explain why the formula represents a linear scenario, create an appropriate formula, and analyze the key features (slope and intercepts) of the linear function.

Rationale for selecting/designing this problem/task sequence:

- This problem was selected because it discusses key aspects of linear functions through an application that all students can relate to since everyone has been a child and/or has a child/children.

Prerequisite Knowledge:

- Students can create formulas based on scenarios.
- Students identify linear functions as functions with a constant rate of change.
- Students can identify a constant rate of change by repeated addition or subtraction in a given scenario.
- Students can solve linear equations.
- Students can describe vertical and horizontal intercepts as points on the x-axis or y-axis of the graph.

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Students can describe why a scenario matches a linear function. (SLO 1, 2)
- Students can write a linear formula based on a scenario. (SLO 1, 2)
- Students can find vertical and horizontal intercepts algebraically. (SLO 1, 2)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Linear functions

Targeted understanding of the key idea/topic:

- For students to recognize the slope of a linear function as successive, equal changes in the input quantity corresponding to consistent changes in the output quantity
- For students to recognize an initial value as the constant term of a linear function
- For students to make connections between multiple representations of the same linear function for various properties, such as slope, initial value, and x-intercept

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
    - Transitional understanding vs targeted understanding

Constant ROC implies linear function	Identify 0.5 inches each month as a constant ROC

↓ Explain that this function is linear.

Initial value implies y-intercept	Identify 0.5 inches per month as slope and 20.5 inches as initial value.	y = mx + b

↓Students can write a linear formula based on a scenario.
↓Students can graph the linear function.

Students can describe vertical and horizontal intercepts.

↓Find (0, 20.5) as vertical intercept.
↓Find (-41, 0) as a horizontal intercept.
↓*What do the intercepts tell us about this scenario?*

Discuss intercepts in terms of context.

Problem Instructor Guiding Questions:

My son Daniel was 20.5 inches when he was born. Suppose he grows approximately 0.5 inches each month.

a) Explain why the function showing Daniel's height as a function of the number of months is a linear function. If students struggle, instructors can ask questions such as "Is there

repetition in the problem?” “Is the repetition constant or changing?” “How would you calculate his height in a specific month?”
Most students have an understanding of linear functions as equations of the form y = mx + b and graphs are linear (from previous math courses going back to pre-algebra). Focusing on the repetitive nature of the scenario would be best to understanding the function of linear.

b) Write a formula for this function. If students struggle, instructors may suggest using a table or graph (something they are usually more familiar with at this point in the class). Instructors may also suggest calculating his height in a specific month (or several) to provide some math with the problem, which can be generalized to a formula. Students generally remember y = mx + b as linear so you might use that as a hint for struggling students.

c) Use functional notation to show Daniel’s height at 36 months, and then calculate that value.
*You could change units on this one and ask for his height at 3 years (making sure students are checking units). This is a great time to point out how a linear function is only appropriate for a short amount of time with growth and that a typical child’s growth rate shows an increasing at a decreasing rate pattern (rather than a constant rate).

d) Make a graph of Daniel’s height function.

e) Identify the vertical intercept (y-intercept) and slope of the graph. Explain in practical terms what they mean.

f) Find the horizontal intercept (x-intercept) and explain its meaning in practical terms.
*This question is nice because the horizontal intercept doesn’t have a very practical meaning. Students need to know that sometimes it does and sometimes it doesn’t. We can always find it though.
*Students have been calculating and interpreting slope and intercepts since Algebra 1.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- Students use multiple representations throughout the problem by creating an equation and graph. Students may benefit from also making a table to help them formulate the graph or equation. Students get to pick which representation makes most sense to them. Students select the type of function. Students relate the graph and equation key characteristics to evaluate their meaning.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The student scenario has been shortened and made more open-ended to allow for more student discussion.
- Students select a function to represent the scenario.

Optional extensions of the problem.

- Students could look up information about their height at birth and height at another age. Then create an equation for themselves. Compare this with Daniel's information. This is information provided that could be used (from other members of the group).
    - Grayson 7 lbs 12 ozs and 20.5 inches
    - Cayden 4 lbs 15 ozs and 20 inches
    - Quinn 6 lbs 3 ozs and 22 inches
- Students could research growth charts for animals.
- This problem could be extended to include weight or head circumference.

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- The growth of 0.5 inches each month gives the constant rate of change structure in an easily accessible context. This context is something that students can relate to with anything else that is growing/increasing (tree, rain, water in a pool, etc.)

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- N/A

Optional extensions of the problem.

- This problem could be extended with another child's information given in different forms (maybe with two points). The students could create another equation and compare the two children's slope, y-intercept, and x-intercept.
- By extending the problem to include weight or head circumference, then the students are provided an opportunity to generalize the material.
- This problem could be expanded to include all 4 representations - table could be added. The use of Desmos technology would be a good way to show the multiple representations in the classroom.
- Extend this problem for growth rates based on different ages (domains) to a piecewise function.
- Other extension contexts could be gaining a steady weight during the freshman year of college, increasing a basketball score by a certain amount of points per minute, decreasing water in a tank by a certain amount per minute (or emptying a pool).

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- Students use academic success skills by problem solving with an application problem that should be of interest to them. Students must persevere through the problem because they are working with a partner or group rather than being given all material by the instructor. Students think critically about the scenario, how to write the formula, and the relationship between key characteristics on the equation and graph.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- Problem was re-written to be more open with the idea that students would work together in groups and more discussion would occur (more student-centered versus teacher-led). This helps build a sense of belonging in a mathematical community.

Optional extensions of the problem.

- With the extensions described above, students will use more problem solving and critical thinking skills through interest and engagement. By using their own data (or their children's data, etc.), the students will be motivated through the curriculum to learn the material.