Problem Name/Description: Exponential with Input Change (not equal to 1)

In this problem, students must create an exponential model given two points.

https://healthcostinstitute.org/diabetes-and-insulin/spending-on-individuals-with-type-1-diabetes-and-the-role-of-rapidly-increasing-insulin-prices.

Rationale for selecting/designing this problem/task sequence:

- This problem is at the end of the exponential module because it isn't presenting new learning objectives necessarily. This problem gives students the opportunity to take what they've learned about exponential functions and apply it in a way that they maybe haven't done yet.

Prerequisite Knowledge:

- Exponential functions have a constant multiplier over consistently-spaced intervals.
- Exponential functions are of the form $f(x) = ab^x$, where a is the starting value and b is the growth/decay factor.
- Solve nonlinear equations using calculator techniques.

Learning objective(s) and alignment with Course objectives (CLO From CEP Matrix)

- Students can apply information about exponential functions to applications. (CLO 1, 2)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Exponential Function Equations

Targeted understanding of the key idea/topic:

- For students to reason about an exponential constant multiplier within the context of a change in the input quantity other than one unit

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
    - Transitional understanding vs targeted understanding

Exponential functions have a constant multiplier over consistently-spaced intervals.

↓ *There are four years between 2012 and 2016, but there are five years between 2016 and 2021.*

An exponential function equation is needed for this scenario.	Exponential functions are of the form $f(x) = ab^x$, where a is the starting value and b is the growth/decay factor.

↓ *Identify cost of insulin (in dollars) as the function outputs.*
↓ *Identify years (aligned to 2012) as the function inputs.*

Recognize there were two points given: (0, 2864) and (4, 5705)

↓ *Recognize f(0)=a.*

$f(x) = 2864 \cdot b^x$

↓ *Plug in the point for 2016*

$5705 = 2864 \cdot b^4$	Solve nonlinear equations using calculator techniques.

↓ *Solve for b using Solver or Intersection of Graphs methods.*

$f(x) = 2864 \cdot 1.188^x$

↓ *Identify 2021 as an input of 9.*

↓ *Input 9 into f(x).*

$$f(9) = \$13,500.78$$

Problem Instructor Guiding Question:

From 2010 to 2020, the cost of insulin increased exponentially. According to the Health Care Cost Institute, annual per capita insulin costs for diabetics with employer-sponsored insurance averaged $2,864 in 2012; by 2016, average costs had climbed to $5,705.

(a) Write an exponential model for the annual cost of insulin for a person with diabetes, with inputs being the number of years since 2012.
(b) Based on the model from part (a), estimate the annual per capita cost of insulin in 2021.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it.

- The task supports students' covariational reasoning because it requires careful attention to both the input and the output quantities, both in identifying the ordered pairs given and in finding the constant multiplier of the exponential function.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The task was initially scaffolded into two parts. This scaffolding was removed to make the problem more open so that students could select the method that makes the most sense to them.

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it.

- N/A: This is meant as an end-of-unit problem that is more challenging, not an extension of anything easily accessible to students.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it.

- This task contains a link to the source of this data. Even if students are not interested in insulin prices, they are seeing an example of how exponential functions are relevant to society.
- This task relies on previous understanding of exponential functions. Students are encouraged to look through their previous notes to find relevant information when brainstorming.
- This task can be done in groups in the classroom, which is helping build a sense of community.

Optional extensions of the problem.

- Students can select a context that is important to them that is changing rapidly. Students find two data points for their context and predict future outcomes if the exponential trend continues. This empowers students to utilize the mathematical concepts in a real-life application that they identify with.