Problem Name/Description: Applying Exponential Ideas

In this problem, students must use the ideas of exponential functions to work in reverse to understand an exponential scenario.

https://tasks.illustrativemathematics.org/content-standards/tasks/533

Note: This problem was pulled from a credible source and mostly serves as an example to apply the framework.

Rationale for selecting/designing this problem/task sequence:

- This problem is highly conceptual about exponential functions and how they work. This problem is application based.

Prerequisite Knowledge:

- Students can identify exponential data or trends based on scenarios.
- Students can write an equation of an exponential function using initial value and growth/decay factor.

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Students can identify growth or decay factors based on scenarios. (SLO 2, 3)
- Students can apply information about exponential decay to applications. (SLO 1, 2, 3)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Exponential Functions

Targeted understanding of the key idea/topic:

- For students to recognize the constant multiplier of an exponential function as successive, equal changes in the input quantity corresponding to consistent multiplicative changes in the output quantity
- For students to reason through a constant multiplier way of thinking, but with decreasing input values instead of increasing input values

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
  - Transitional understanding vs targeted understanding

Students can identify exponential data or trends based on scenarios.	Students recognize a growth factor of 2 means they should divide by 2 when working backwards.

↓ *Students answer part a by starting on June 30th and going backwards to June 29th.*
↓ *Students answer part b by continuing the process from (a) until they reach June 26th.*

Students are able to work backwards discretely in exponential situations.

↓*Students start with 1% at the new "Day 0".*
↓*Students discretely double each day until 100% is reached.*

Students answer part c by recognizing that 1% will double back to 100% very quickly.	Students can recognize the growth factor as 2 in doubling situations.	Students can find an initial value by using a point and growth/decay factor.

↓$y = a(2^{x})$
↓ *Students recognize June 1st as Day 0, June 2nd as Day 1, … , June 30th as Day 29.*
↓$100 = a(2^{29})$
↓*Solve for a*

Students can write an equation of an exponential function using initial value and growth/decay factor.

↓Students write equation for part d.

Problem Instructor Guiding Question:

On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

a) When will the lake be covered half-way?

*Usually students either get this fairly quickly or need decent hints. Try to emphasize the idea that the algae is doubling everyday, but to think backwards on this problem. If that isn't enough, then ask if everything is covered, how much is covered the day before? This is almost a give-away at this point, but some students need it.

b) On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?

*Generally students try using a table or list of values on this because they want to work backward (utilizing their ideas from part a) to find out the fraction (or percent) covered for June 26.

c) On June 29, a clean-up crew arrived at the lake and removed almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

*At this point, students usually realize that the exponential doubling effect is going to undo the 1% really quickly.

d) Write an equation that represents the percentage of the surface of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake.

*This is the trickiest part of this problem for most students. They know the multiplier is 2, but must either work backward or plug in a point and solve by crossing graphs to find the initial value. This part generally works better as a full class discussion.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- The open-ended questions provide an opportunity for students to select, perform, and evaluate actions. Students must "try" and experiment with different strategies. Students must "argue" through the question process.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- N/A

Optional extensions of the problem.

- Revise/add questions with Bloom's taxonomy in mind

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- Students extend prior knowledge of exponential material to a new application with critical thinking questions.

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- N/A

Optional extensions of the problem.

- Add questions and extend the material to logarithmic functions to emphasize the inverse relationship between exponential and logarithmic functions.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- Students must persevere through higher order thinking questions that emphasize conceptual understanding of exponential functions.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- N/A

Optional extensions of the problem.

- N/A