Problem Name/Description: Constant Percentage Change

In this problem, students will transition into understanding exponential functions with a constant percentage change and how that relates to the constant multiplier (previously discussed).

Rationale for selecting/designing this problem/task sequence:

- This problem introduces students to exponential functions using vocabulary such as constant percentage change (rather than constant multiplier).

Prerequisite Knowledge:

- Calculate percentages.
- Multiplication/Division
- Write an exponential model given an initial value and growth/decay factor.

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Calculate percent change.
- Understand percent change as the ratio between the change and the original value, converted to a percentage.
- Identify a constant percent change with a constant multiplier. (p%=(b-1)100%) (SLO 2, 3)
- Identify a constant multiplier with an exponential function. (SLO 2, 3)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Exponential Functions

Targeted understanding of the key idea/topic:

- For students to recognize the constant percent change of an exponential function as successive, equal changes in the input quantity corresponding to consistent percent changes in the output quantity
- For students to develop the relationship between a constant multiplier of an exponential function and the constant percent change.

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
    - Transitional understanding vs targeted understanding

Calculate percentages.

↓ *Find percent by which account balance increased in the first week.*

25% increase	Calculate percentages.

↓*Find account balance for the first five weeks.*

Multiplication/Division

↓*When looking at the account balances, what constant could you multiply by to get the next week's balance?*

1.25

↓*How is this related to the 25% increase?*

1.25=1+ 25/100

↓*Write a function equation that will give the account balance with respect to the week.*

$f(x) = 20(1.25)^{x}$

Problem Instructor Guiding Question:

Suppose your aunt opens a bank account for you and deposits $20 into it at the beginning of the semester.

(a) She says that if you have a grade at or above a C in MATH 1643 after the first week of the semester, she will deposit an additional $5. Assuming that you are successful, what is the account balance after week one? By what percentage did the account balance increase from the beginning of the semester to the end of week one?

(b) Suppose you and your aunt make a deal that if you continue to have a grade at or above a C in MATH 1643, each week she will make a deposit equal to 25% of the account balance.

In anticipation of your hard work and success, fill out the table and calculate your account balance throughout the semester. Note that this increase in money is a ***constant percentage change*** each week.

Week	Weekly Deposit (dollars)	Account Balance (dollars)
0	20	20
1	5	25
2		
3		
4		
5		

When looking at the account balances, what constant could you multiply by to get the next week's balance? ***How is this related to the 25% increase?*** What constant would you multiply by to get the balance two weeks later?

*This question is to emphasize the relationship between a constant multiplier and the input interval length.

*This is a good opportunity to emphasize what the input and output quantities are and the relationships between them.

(c) Write a complete exponential model for the account balance:

- In the scenario with your aunt, what is the starting balance?
- What is the constant multiplier, b, and what time period does it correspond to?

*Could also have students use their model to check the relationship between their constant multiplier and constant percentage change.

(d) (optional) Suppose you make it to the end of finals week maintaining a grade at or above a C. What is the account balance at the end of the semester? (Assume finals week is the 16th week of the semester.)

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- Students create a table to analyze constant percentage change.
- Students work back through their table to identify how they could have created it from multipliers instead of percentages.
- Students use multiple representations.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The student scenario has been shortened and made more open-ended to allow for more student discussion. Important topics of discussion include the relationship between constant percentage change and a constant multiplier, the relationship between the constant multiplier and input interval length, and the relationships between the starting value, constant multiplier, and exponential equation.
- Students select strategies for how to work through the problem rather than being told to make a table, etc.

Optional extensions of the problem.

- Expand the problem to include real or play money.
- Provide multiple scenarios with various percentages. Ask students to make predictions of what scenario would have the most money after a certain time period. Compare with the other groups.
- Reverse process. Find what percentage of the original amount is _____.

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- The problem utilizes percentages and money to build to the idea of a constant percent change.
- Students make claims about how the percentage change is related to the constant multiplier. They confirm the claim with the mathematical model.

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- Problem has been restructured so that students create their own table for the first five weeks and reconsider their table in terms of a constant multiplier instead of a constant percentage change. The constant multiplier is prerequisite knowledge with a parallel structure to a constant percentage change.

Optional extensions of the problem.

- Students build on ideas of percentages, percent change and finances.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- Students work with a real-life problem involving finances, money, and percents to help with their financial literacy.
- Context promotes good academic success skills via the idea of financial compensation.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- Problem was re-written to be more open with the idea that students would work together in groups and more discussion would occur (more student-centered versus teacher-led). This helps build a sense of belonging in a mathematical community.

Optional extensions of the problem.

- Have students research exponential functions related to their field of study that involve a constant percentage change.
- Suppose your aunt continues to make a 25% deposit every week, even after the semester has ended. What will the account balance be after one year? (Instructor: Opportunity here to discuss how fast the balance grows since it's exponential.)