Problem Name/Description: To the Moon with Exponential Functions

In this problem, students will transition from linear functions to exponential functions through paper folding to the moon (and back). Students will see how exponential functions increase at an increasing rate and decrease at a decreasing rate through the use of tables (and a graph in the video). Students will explore standard form of an exponential function by creating two models based on the paper folding scenario.

Rationale for selecting/designing this problem/task sequence:

- This problem was selected to emphasize the difference between linear and exponential functions. This can be used for an introduction to exponential functions and is something that students can relate to. Many students have had the challenge of folding paper as many times as possible.

Prerequisite Knowledge:

- Students can create formulas based on scenarios.
- Students can create a table based on scenarios.
- Students can use appropriate variables and units for scenarios.

Learning objective(s) and alignment with Student Learning Outcomes (SLO From CEP Matrix)

- Students can discriminate between exponential and linear functions. (SLO 1, 2, 3)
- Students can identify exponential data based on a table/pattern. (SLO 1, 2, 3)
- Students can create exponential equations based on scenarios. (SLO 1, 2)
- Students can describe what the numbers in an exponential equation mean in terms of scenarios. (SLO 1, 2)
- Students can discriminate between exponential growth and exponential decay (SLO 1, 2, 3)

Identify the key idea/topic that would be the subject of the conceptual analysis:

- Exponential Functions

Targeted understanding of the key idea/topic:

- For students to recognize the constant multiplier of an exponential function as successive, equal changes in the input quantity corresponding to consistent multiplicative changes in the output quantity

- For students to recognize the constant multiplier in the context as the base value of an exponential function
- For students to make connections between multiple representations of the same exponential function for the constant multiplier and initial value

Conceptual Analysis (HLT):

- *How* are the prerequisite topics relied on or used in order to reach the learning objective? (conceptual analysis)
- What are the milestones of a task sequence? (Hypothetical learning trajectory)
    - Transitional understanding vs targeted understanding

Students create a table based on the thickness scenario.

↓ *The thickness doubles with every fold.*
↓ *The previous thickness needs to be multiplied by 2.*

Students identify that the scenario is not linear because there is a multiplier (percentage rate of change) rather than constant rate of change.	Students use appropriate variables and units.

↓*Students recognize repeated multiplication as an exponent.*

Students create a formula based on the thickness scenario.	Students create a table based on the area scenario.

↓ *The area is cut in half with every fold.*
↓ *The previous area needs to be divided by 2.*

Students identify that the scenario is not linear because there is a multiplier rather than constant rate of change.	Students use appropriate variables and units.

↓*Students recognize repeated division as an exponent with a fractional base.*

Students create a formula based on the area scenario.

↓*Students compare the thickness and area scenarios and formulas.*

Students identify exponential growth with a base larger than 1 and exponential decay with a base smaller than 1, when input intervals are 1-unit.

Problem Instructor Guiding Question:

Watch the Paper Folding Video together in class. (Pause throughout the video to point out key ideas of the problem and its relationship to exponential functions)

We recommend the following pauses:

At 0:17 - Ask students if they have ever done a paper folding challenge. Most have some experience with folding paper and say they can only fold it 7ish times.

At 0:54 - Ask the students if the thickness of the paper would have a big impact or not on paper folding.

At 1:11 - Ask the students how thick they think paper would be after 30 folds. Sometimes they don't know where to start so you might ask how the thickness compares to classroom items such as a binder, the desk, the ceiling, the building, etc. Typically students think 30 folds are no thicker than a desk height.

At 1:34 - Ask students what they notice about the numbers (they should notice they double each time).

At 2:28 - Pause for impact! Students are taken by surprise at how quickly the thickness grew.

At 2:52 - Discuss the term exponential growth. Ask students how the graph compares with linear functions. Ask the students about concavity and what it means about how the graph increases.

At 3:33 - Ask if anyone knew it was that easy to go to the moon.

In the video, you saw that the paper had an original thickness of 0.001cm and doubled in thickness with each fold.

a) Complete the table to show the thickness up to 5 folds.
*It is nice to show the work on the table in terms of 0.001 and 2 (rather than just showing answers). For example, for 3 folds, we recommend writing 0.001 (2) (2) (2) so that the students can see the repeated multiplication (exponents).

Folds	0	1	2	3	4	5

Thickness						

b) Write a formula for thickness based on the number of folds to represent what happens in this scenario. Use n for number of folds and T for the thickness of the paper in centimeters.
*If students are struggling with the formula, make sure they have followed the suggestion from part a. If they have, then ask what mathematical operation shows repeated multiplication. That is generally the key connection to make to form an equation.

c) What makes this scenario exponential instead of linear?
*This should come from the video along with the equation. In the video, exponential growth is discussed. When students look at the equation, they should see exponents rather than y = mx + b.

d) Consider what would happen if you measure the area of the paper with each fold instead of the thickness. Complete the table to show the area up to 5 folds if you start with a paper that has an area of 93.5 square inches (standard computer paper).
*We recommend showing work based on 93.5 and ½ (since the area is shrinking by ½).

Folds	0	1	2	3	4	5
Area						

e) Write a formula for area as a function of the number of folds to represent what happens in this scenario. Use n for number of folds and A for the area of the paper in square inches.

f) What makes this scenario exponential instead of linear? Compare and contrast this scenario to the thickness problem.
*This introduces the idea of exponential decay to students. They should still note there is a starting value along with a multiplier.

g) How many times do you need to fold the paper until it fits on a dime? (A dime is 0.39040 square inches).
*This would be done with crossing graphs.

Active Learning:

Evaluation of the extent to which this task engages students in active learning as the MIP has defined it

- During the video, the instructor pauses the video multiple times for classroom discussion. Instructor allows students to make predictions (such as how thick the paper will be after a certain number of folds and how many folds it will take to get to the moon). Students compared prior knowledge of linear functions with this new idea/concept.

Changes that have been made to make the task more aligned with active learning as the MIP has defined it.

- The student scenario has been reordered and made more open-ended to allow for more student discussion.

Optional extensions of the problem.

- Have students fold their own paper, collect data, and perform exponential regression.
- Have students research where else they see exponential functions.

Meaningful Application:

Evaluation of the extent to which this task engages students in a meaningful application as the MIP has defined it

- Students are introduced to the difference between linear growth and exponential growth.

Changes that have been made to make the task more aligned with meaningful applications as the MIP has defined it

- The extension to area has been more emphasized so students can compare exponential growth versus exponential decay.

Optional extensions of the problem.

- Have students research where else they see exponential functions.
- Extend to multiple representations. Use Desmos for tables and graphs.

Academic Success Skills:

Evaluation of the extent to which this task engages students in academic success skills as the MIP has defined it

- Students reflect on how exponential functions differ from linear functions.

Changes that have been made to make the task more aligned with academic success skills as the MIP has defined it

- Problem was re-written to be more open with the idea that students would work together in groups and more discussion would occur (more student-centered versus teacher-led). This helps build a sense of belonging in a mathematical community.

Optional extensions of the problem.

- Students compare and contrast linear and exponential in more detail with multiple representations.

- Have students research where else they see exponential functions in their field of study.