Option 1: Large Group Activity

The table gives the population of a town starting in 1970. The total population *P* is given for the number of years *n* after 1970.

Number of years after 1970 *n*	0	5	12	17	24
Total population *P*	2350	2575	2866	3124	3359

a) Identify *P(0)*. Describe, in words, what this means in terms of the population. Make sure to include appropriate units.

$P(0) = 2350$ people
This is the population, in people, 0 years after 1970 (or, in other words, 1970).

b) Describe, in words, the changes in population per year for the values given in the table. Show any calculations to support your description.

Students may just see a general increase in the population, however, a more thorough description of AROC is below (and may be used for part b or for part c).

Since this table does not have evenly spaced values for the number of years, we will need to consider the AROC for each interval provided.

0 to 5 years: $\frac{2575-2350}{5-0} = \frac{225}{5} = 45$ people per year

5 to 12 years: $\frac{2866-2575}{12-5} = \frac{291}{7} = 41.6$ people per year

12 to 17 years: $\frac{3124-2866}{17-12} = \frac{258}{5} = 51.6$ people per year

17 to 24 years: $\frac{3359-3124}{24-17} = \frac{235}{7} = 33.6$ people per year

c) For which interval was the population increasing most rapidly? Least rapidly? Make sure to include appropriate units for any mathematics.

The population increases most rapidly in the time interval of 12 to 17 years and least rapidly in the time interval of 17 to 24 years.

It is good to point out that this is an average increase. We would not see an exact increase of 33.6 people per year. Generally, we round people to the nearest whole number.

d) Given an AROC of 33.6 people per year, what does that mean in terms of the population? Would you expect the population to increase by exactly 33.6 people each year?

For the AROC of 33.6, we would expect an increase of 33.6 people each year. However, since we are dealing with people, we would generally round up to 34 people each year.

e) Does the table make you think that the population has a limiting value? Support your answer. Describe what happens with AROC if there is a limiting value for a given set of values.

No, this does not support a limiting value because the general trend of the graph is to continually increase in population. It is not likely to level off at a specific value based on the given data.

For a table to have a limiting value, the AROC must decrease to 0.

f) Estimate $P(9)$ and $P(19)$. Can you find more than one way to estimate these values? Did you use the same method for each estimation? Explain. Make sure to include appropriate units.

$P(9)$ Option 1: Students may average the values from the population at year 6 and the population at year 12. This would most likely be thought of as finding the middle value (rather than thinking of the mathematical average).

$\frac{2575+2866}{2} = 2720.5$

$P(19)$ Option 2: Students may use the average rate of change (48.5 from part b) to find the middle value.

Sample calculations may be $2575 + 48.5(3)$ or $2866 + 48.5(-3)$.

In addition, students may use other values like population levels at years 0 and 18 to find the average rather than years 6 and 12.

g) Consider the following calculations. Describe what mathematics is being shown and what it is being calculated in practical terms.

i) $2575 + \frac{2866-2575}{12-5}(5)$

- 2575 is the population at year 5 or P(5)
- $\frac{2866-2575}{12-5}$ finds the average rate of change between years 5 and 12
- Multiplying it by 5 is calculating the total increase over 5 years after year 5

The total represents P(10) the population 5 years after year 5,
P(10) = P(5) + 5(average rate of change from year 5-12)

ii) $2866 + \frac{2866-2575}{12-5}(-2)$

- 2866 is the population at year 12 or P(12)
- $\frac{2866-2575}{12-5}$ finds the average rate of change between years 5 and 12
- Multiplying it by (-2) is calculating the total decrease over 2 years prior to year 12

The total represents P(10) the population 2 years prior to year 12,
P(10) = P(12) - 2(average rate of change from year 5-12)

h) Using the data given, how would estimate $P(29)$? Show your calculations and explain this process.

Over the 24 year period, the population continued to increase but was at an increasing rate for the first 12 years then a decreasing rate for the next 12 years. If the decreasing rate continued P(29) could be estimated using the rate from the interval 17 - 24 years

17 to 24 years: $\frac{3359-3124}{24-17} = \frac{235}{7} = 33.6$ people per year

P(29) = year 24 population + 4(avg. yearly population) = 3359 + 4(33.6) = approximately 3493