

## Lecture Notes – Weighted Averages

### Recall Using a Frequency Table

**Example:** Suppose a class of 10 students received the following scores on an assignment as shown in the frequency table below.

Score (%)	Frequency
68	2
72	3
75	1
83	4

Notice that if we want to find an average score obtained by the students in the class, we need to take into account not only the scores themselves, but also the frequency of the scores. This means we will be computing a weighted average. Since the score of 83% has a frequency of 4, it will contribute more to the overall weighted average than the score of 75%, which only occurs once.

Let's review how to compute the (weighted) average score.

*Note:* The last row is the column totals.

Score ( $x$ )	Frequency ( $f$ )	$xf$
68	2	136
72	3	216
75	1	75
83	4	332
	<b>10</b>	<b>759</b>

Weighted average =  $759/10 = 75.9\%$

This leads us to the definition of a weighted average.

### Definition

A weighted average is a type of average that takes into account the varying degrees of importance of quantities and hence some quantities contribute more than others to the final result.

### Back to the Example

In the above example, the “weights” were the frequencies of the scores. Oftentimes, the weights are represented as decimals, where the sum of the weights is equal to 1 (or 100%).

Let's create a chart that represents the above information using equivalent weights written as decimals.

Notice that there are 10 students in the class and hence there were 10 total scores. The score of 68% occurred 2 times. Thus, its “weight” is  $2/10 = 0.2$ . Similarly, the score of 72% occurred 3 times. Thus, its “weight” is  $3/10 = 0.3$ . Continue in this manner to complete the chart.

Score (%)	Weight (decimal)
68	0.2
72	0.3
75	0.1
83	0.4

We compute the weighted average using the same method as we did previously. The third column, the weighted scores, will be computed by multiplying the score by its corresponding weight. Each weighted score gives the number of points contributed to the overall weighted average. Again, the last row has the column totals.

Score (%)	Weight (decimal)	Weighted Score
68	0.2	13.6
72	0.3	21.6
75	0.1	7.5
83	0.4	33.2
	<b>1.0</b>	<b>75.9</b>

Thus, the weighted average score is:  $75.9/1.0 = 75.9\%$ . This is the same answer we computed above! Yay!

Notice that each score contributes points to the overall weighted average score. The amount of points that are contributed is dependent on both the score and its weight. For example, the score of 83% has a weight of 0.4 which gives a weighted score of 33.2. In turn, this score will contribute 33.2 points to the overall weighted average score.

**Let’s show why these two methods are equivalent**

$$\begin{aligned}
 & \frac{68(2) + 72(3) + 75(1) + 83(4)}{2 + 3 + 1 + 4} \\
 &= \frac{68(2) + 72(3) + 75(1) + 83(4)}{10} \\
 &= 68\left(\frac{2}{10}\right) + 72\left(\frac{3}{10}\right) + 75\left(\frac{1}{10}\right) + 83\left(\frac{4}{10}\right) \\
 &= 68(0.2) + 72(0.3) + 75(0.1) + 83(0.4) \\
 &= \frac{68(0.2) + 72(0.3) + 75(0.1) + 83(0.4)}{1}
 \end{aligned}$$