# Activity 2: Competing Social Media Platforms

## Task: Competing Social Media Platforms – with instructor notes

The tasks in this module are designed to support students' in developing quantitative, covariational notions of concavity. These tasks are designed from the perspective that concavity is not an end to itself: identifying intervals on which a function is concave up and concave down is useful because it provides information about the relationship between the input quantity and output quantity. That is, rather than perceive concavity as an attribute of the shape of a function's graph – which, on its own, provides no insight into how the input and output quantities change together – these tasks focus on developing understandings of concavity in terms of *amounts of change* and the *average rate of change* (see Carlson et al., 2002).
These tasks rely on the following prerequisite ideas and skills:

- Understanding a function as a description of how two quantities change in relation to each other
- The ability to view and interpret functions across the various representations of functions: formula, table, graph, and words
- Understanding the average rate of change as the rate at which a function is increasing (i.e., on a given input interval, the average rate of change is the constant rate of change needed to yield the corresponding change in output on that input interval)

These tasks focus on viewing concavity in terms of the relationship between changes in the input quantity and changes in the output quantity. Specifically:

- We want students to associate "concave up" and "concave down" as a statement about how the input and output quantity change together. This can be productively understood via amounts of change and the average rate of change:
    - *Amounts of change:* to say that a function is "concave up"/ "concave down" on an interval indicates that successive, incremental increases in the input quantity within the given interval correspond to increasing/decreasing changes in the output quantity
    - *Average rate of change:* to say that a function is "concave up" / "concave down" indicates that successive changes in the input quantity within the given interval correspond to an increasing/decreasing average rate of change
- We want students to be able to be able to enact these understandings to recognize and reason productively about concavity across each of the various function representations. For example, if a student recognizes that a function is concave up on a particular interval using the "up like a cup" rule, they also (1) know that this provides useful information about the relationship between the input and output quantities, and (2) are able to identify and articulate what this additional information is.

**Learning Trajectory:**

- ***Phase 1 (highlighting the importance of amounts of change and/or the average rate of change)***: Students are not explicitly directed to employ notions of amounts of change or the average rate of change; rather, they emerge in students' reasoning as useful tools for answering the questions that have been posed. Initially leave the approach to solving the problem open for students to identify, attempt, and evaluate their own ideas and methods. Facilitate discussion and sharing to encourage students to compute the average rate of change for each interval.

- ***Phase 2 (using the behavior of the amounts of change and/or the average rate of change):*** *Reasoning about the values of the input and output quantities alone will not be sufficient to answer the question*. Facilitate discussion and sharing to encourage students to attend to corresponding amounts of change and the average rate of change;

- ***Phase 3 (extending the quantities and relationships identified in Phases 1 and 2 to view them from other conventional function representations)*:** A table representation lines up the amounts of change and average rates of change along successive subintervals in the same column so that the fact that they are increasing/decreasing can be more readily observed. A graphical representation enables students to connect the increasing/decreasing amounts of change and average rate of change to the more conventional "up like a cup" and "down like a frown" associations with concavity. We note, however, that this exercise is meaningful for students if and only if the behavior of the amounts of change and the average rate of change emerged in their own reasoning across Phases 1 and 2.

- ***Phase 4 (providing the formal names and definitions for concave up and concave down)*:** Use the two questions provided to help students connect their informal reasoning and strategies to the names and definitions of the formal concepts.

- ***Phase 5 (extending these notions of concavity to other scenarios and contexts):*** Extending beyond the task setting in this Phase is designed to strengthen their emerging understandings in two ways. First, it will refine their understandings, for example encouraging students to grapple with concavity when input intervals are not pre-determined or unequal. Second, it will broaden the scope of the understandings they developed in Phases 1-4, helping to ensure that their understandings are not tied to any one particular task setting.

**Active Learning:** these tasks are non-routine and somewhat open-ended. Possible productive approaches include: comparing changes in a quantity across multiple intervals, using the "most recent" amount of change or average rate of change to make a prediction, or making inferences about an amount of change or average rate of change is going to be in a subsequent interval based upon the observation that it is increasing/decreasing along previous intervals.

**Meaningful Applications:** Though contrived, this application supports students in identifying key mathematical patterns involving the increasing/decreasing nature of the average rate of change and then interpreting what this means in terms of the relationships between the input and output quantities. Additionally, these tasks – in particular, Phase 5 – encourage students to generalize the notion of concavity and the behavior of the average rate of change to other contexts.

**Academic Success Skills:** these tasks seek to develop students' conceptions of their own role as active contributors in the mathematics classroom as well as their conceptions of mathematics as a creative, scientific endeavor that involves generating, testing, and revising one's intuitive hypotheses.

**Activity: Competing Social Media Platforms**

[*Note to instructors*: this is not intended to be given to students all at once. Using a combination of group work and class discussion, the class should complete each phase in its entirety before moving on to and introducing the next one. See the notes above for additional recommendations for classroom implementation.]

Suppose that you are a writer for a tech blog and are comparing two new social media platforms – InstaTwit and SnapFace – both of which launched at the same time and are cutthroat rivals. It is now 24 weeks since both platforms have launched, and you are hoping to predict which of the two will reach 200 million total users first.

For InstaTwit, you find the following information:

- After the first 8 weeks of its existence, InstaTwit had 24.2 million total users.
- 16 weeks after launching, InstaTwit had 68.28 million total users.
- 24 weeks after launching, InstaTwit had 111.32 million total users.

For SnapFace, you find the following information:

- During the first 8 weeks of its existence, SnapFace added an average of 7.09 million new users per week
- During the next 8 weeks, SnapFace added an average of 5.54 million new users per week
- During the next 8 weeks, SnapFace added an average of 2.33 million new users per week

**Phase 1.** Who has more users after the first 24 weeks?

**Phase 2.** Make a prediction about which website will be the first to reach 200 million total users. Write a sentence explaining your prediction ("we believe that _______________ will reach 200 million total users first because …").

**Phase 3.** Consider the total number of InstaTwit users $I$ (in millions) as a function of the number of weeks $w$ since their platform launched. Similarly, consider the total number of SnapFace users $S$ (in millions) as a function of the number of weeks since their platform launched. Use the rationale you based your prediction on to represent the behavior of the total number of users for InstaTwit $I$ and SnapFace $S$ over time in a graph and in a table:

(a) Use your predictions (and rationale for these predictions) to complete the following tables:

<table>
<tr><th>Length of time period (in weeks)</th><th>Number of weeks $w$ since the platform launched</th><th>Total number of InstaTwit Users $I$ (in millions)</th><th>New InstaTwit users added during this time period (in millions)</th><th>Average number of new InstaTwit users added per week during this time period (millions of new users per week)</th></tr>
<tr><td>---------------</td><td rowspan="2">0</td><td rowspan="2"></td><td>---------------</td><td>---------------</td></tr>
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| | 40 | | | |
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| | 48 | | | |
| | | | | |
| --------------- | 56 | | --------------- | --------------- |

| Length of time period (in weeks) | Number of weeks $w$ since the platform launched | Total number of SnapFace users $S$ (in millions) | New SnapFace users added during this time period (in millions) | Average number of new SnapFace users added per week during this time period (millions of new users per week) |
|---|---|---|---|---|
| --------------- | 0 | | --------------- | --------------- |
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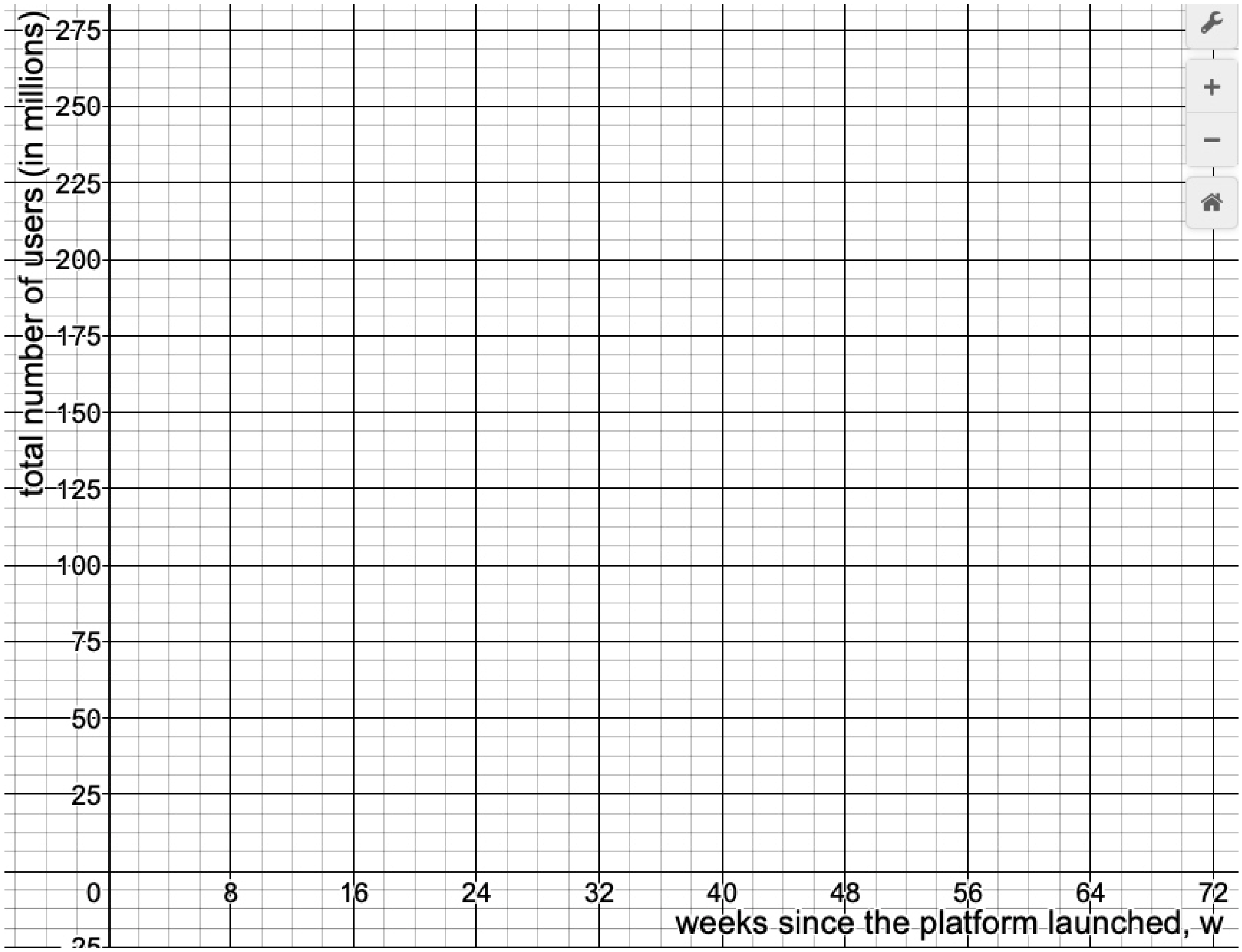
(b) Use the information given above along with your prediction to sketch the behavior of the total number of InstaTwit users and SnapFace users over time:




**Phase 4 (characterizing concavity in terms of the AROC).**

- A function is ***concave up*** on an input interval if its average rate of change is increasing along (successive subintervals within) the interval.
- A function is ***concave down*** on an input interval if its average rate of change is decreasing along (successive subintervals within) the interval.
- A function has ***no concavity*** on an input interval if there is a constant rate of change (i.e. if the function is linear on the input interval).

(a) During the time period depicted in the table and graph, is $I$ concave up or concave down? How can you see this in the table? How can you see this in the graph?

(b) During the time period depicted in the table and graph, is $S$ concave up or concave down? How can you see this in the table? How can you see this in the graph?

**Phase 5 (extending to new scenarios).** In the scenario above, the behavior of the average rate of change (i.e. the concavity) gives us additional information about the behavior of the total number of users for SnapFace (in which case the total number of users is increasing, but the average number of new users added per week is decreasing) and InstaTwit (in which case the total number of users is increasing, and the average number of new users is also increasing). Specifically, we already knew that the total number of users was increasing for both, but the average rate of change gave us information about how it was increasing. In the scenarios that follow, the objective will be to determine if the function is concave up, concave down, or has no concavity on the given interval, and then explain what additional information this provides about the function's behavior.

*Instructions*: For each scenario:

- interpret what the average rate of change represents in this scenario (e.g. the number of new users added per month),
- determine whether the average rate of change is increasing, decreasing, or constant (or perhaps none of these),
- explain what additional insight this provides into the relationship between the given output and the input quantities, and
- create an expanded table representation and a graphical representation and identify how your claims about the average rate of change can be seen in each.

(a) Facebook – the non-fictitious, ubiquitous, all-too-real social media platform – wasn't always an internet giant. As of 2019, there are over 2.3 billion total Facebook users in the world. But 6 months after starting, they had a whopping 0.13 million total users, and it went from there:

| Months since 12/31/2003, $m$ | Total Facebook users (in millions) |
|---|---|
| 6 | 0.13 |
| 12 | 1.0 |
| 24 | 5.5 |
| 36 | 12.0 |
| 56 | 100.0 |
| 59.3 | 130.0 |
| 59.5 | 140.0 |

Consider the total number of Facebook users as a function of the number of months since December 31, 2003.

(b) The total amount of snowfall 3 hours into a blizzard is 3.25 inches. After 6 hours, it's 5.72 inches. After 9 hours, it's 7.11 inches. Consider the total amount of snowfall as a function of the number of hours since the blizzard began.

(c) You return from summer break to find that you owe the university library $235.16 in late fees for a book titled "Philosophical, Microgenetic, and Anthropomorphic Perspectives on the Experiences of Adolescent Mice Living in Southeastern North America in the Mid-to-Late Middle Ages" by Sue Perrbohring. You work out a payment plan with the library: you will pay 15% of the remaining balance each month. Consider the remaining balance $B$ as a function of the number of monthly payments you've made $m$.

(d) The profit $P$ (in thousands of dollars) of a company that produces and sells magic carpets is given by $P(m) = P(m) = -m^2 + 184.9m - 5805.1$, where $m$ is the number of magic carpets produced. Consider only the production interval from 0 to 150 magic carpets.