The Mathematical Inquiry Project

Request for Proposals
Collaborative Research and Development
Quantitative Reasoning

The Mathematical Inquiry Project (MIP) is a statewide collaboration among mathematics faculty in Oklahoma to improve entry-level undergraduate mathematics instruction through three guiding principles:

> ***Active Learning***: Students engage in active learning when they work to resolve a problematic situation whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned. For more information on the MIP Active Learning Principle, visit https://okmip.com/active-learning/
>
> ***Meaningful Applications***: Applications are meaningfully incorporated in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure. For more information on the MIP Meaningful Applications Principle, visit https://okmip.com/applications/
>
> ***Academic Success Skills***: Academic success skills foster students' construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community. For more information on the MIP Academic Success Skills Principle, visit https://okmip.com/academic-success-skills/

Consultation

The MIP encourages those who are interested in submitting a proposal to discuss their ideas with a member of the project team, who can (1) provide feedback and advice on initial ideas, (2) connect potential CoRD members with others who are interested in working on similar ideas, and (3) offer guidance throughout the preparation and execution of the proposal. The MIP team will also organize events throughout the year to allow multiple CoRDs to present their progress and discuss ways to benefit from and integrate their approaches.

Description of CoRD products

The products produced by each Quantitative Reasoning CoRD should contain four features. The MIP team is available to discuss, clarify, and provide resources for each:

- Articulate a conceptualization—informed by the research literature—of the targeted topic (e.g., Ratios, Proportion, and Proportional Reasoning) and describe its relation to or emergence from particular aspects of students' thinking and/or affect.
- Review and synthesize the empirical research literature on the targeted topic that clarifies its importance, identifies and relates its key aspects, demonstrates its significance for students' mathematical development, and documents the effectiveness of instructional interventions (e.g. sequences of instructional tasks and the ideas on which they are based) that have been designed to support students in learning it.

- Propose research-based principles of instructional and/or curricular design to address the targeted topic.
- Explicitly describe how the CoRD's products relate to and/or support the MIP components of inquiry: active learning, meaningful applications, and academic success skills.

After a successful review, the CoRD will test their products with a class or group of students and incorporate a description of the test implementation and its results, a discussion of the refinements and recommendations made based on test implementation, and short video clips with commentary to illustrate effective implementation.

Review and Revision

Once a CoRD submits a module, it will be reviewed by at least two other faculty with expertise in the topic to inform an editorial decision of "accept with minor revision," "revise and resubmit," or "reject," along with directions for revision if appropriate. (This more formal stage of review is distinct from the more informal feedback that a team might receive by interacting with an MIP team member on an ongoing basis.) After a favorable review, the CoRD will revise and pilot their module, incorporating feedback gained during the review process and submit a final module for publication on the project website.

Author Stipends

Each author in the CoRD will receive a $2,500 stipend after delivery of a complete initial draft of the module and an additional $1,000 stipend after delivery of a complete revision of the module based on the editorial decision.

Opportunities for leading regional workshops

The MIP will leverage faculty leadership and expertise developed through its Initiation Workshops and CoRDs to also develop and deliver 40 institutional and regional professional development workshops, across the state of Oklahoma. Each Regional Workshop will last a full day and engage approximately 20 mathematics faculty in implementing one or two of the modules developed by the CoRDs and ensuring familiarity with the module resources. Each workshop will be led by faculty from the respective CoRDs with support from the MIP team.

Targeted Topics for Quantitative Reasoning CoRDs

The MIP seeks to support the development of research-based instructional and curricular design principles on the following targeted topics for the Quantitative Reasoning pathway; each of these targeted topics emerged from the work done by the faculty who participated in the Quantitative Reasoning Initiation Workshop. See the ATTACHED pages for more details about each of these topics (which are listed in no particular order).

1. Information presentation and consumption
2. Ratios, Proportions, and Proportional reasoning
3. Quantification
4. Critical thinking
5. Spreadsheets
6. Modeling
7. Problem solving

Proposal Requirements

Proposals should include each of the following:

1. A cover page designating which of the targeted topics the proposed CoRD will address, names of all proposed CoRD members (2-5 people), their institutions, email addresses, and phone numbers.
2. The CoRD's initial image of how the CoRD will address the four foci outlined in the "Description of CoRD Products" above.
3. A description of prior experience of each CoRD member relevant to their development of the proposed module.

Proposal Length

The full text of a proposal should not exceed 2,000 words.

Proposal Submission

Completed proposals should be emailed to William (Bus) Jaco at william.jaco@okstate.edu. Proposals should be submitted by **Friday, November 1, 2021** for full consideration. The MIP will continue to accept and review proposals after this date, however we strongly encourage discussions with the project team for later submissions to avoid proposing work on topics that have already been assigned a CoRD.

The MIP plans to respond to proposals by early November. During the review of proposals, the MIP may request additional information or modifications before approval. Initial draft of modules to be reviewed will be due **August 1, 2022**.

Quantitative Reasoning: Targeted Topic Descriptions for Collaborative Research and Development Teams

The information below are syntheses of the key aspects of each targeted topic developed by the faculty members who participated in the Quantitative Reasoning Initiation Workshop. These syntheses are intended to serve as a guide for faculty members who are interested in or have already joined a CoRD. The MIP is available throughout the CoRD development process to answer questions and provide feedback on ideas or drafts of CoRD products.

TARGETED TOPIC 1: INFORMATION PRESENTATION AND CONSUMPTION
Responsible citizenship in our modern society is driven by quantitative literacy skills. If a society does not ensure that all are able to think critically when consuming and presenting mathematical ideas, then some will be limited in their social, financial, and employment opportunities. Students with an understanding of various modeling techniques and ways in which information is presented (and mis-presented) are better equipped to be contributing members of society. Being critical consumers of information supports people in analyzing problems and making decisions. Students who are critical consumers of information, and good presenters of information, have healthy skepticism and know when to ask for more information.

Generally, modules developed by a CoRD focusing on Information Presentation and Consumption **should** support students in at least several of the following:

- Analyzing and communicating mathematical ideas using appropriate representations. By "appropriate," we mean both mathematically appropriate and appropriate for the audience.
- Presenting data so that general trends, patterns, and hidden features (e.g., normalization, disaggregation) are apparent
- Attending to ethical considerations in presentation and consumption of information (e.g., not misleading intentionally, identifying misleading information in a given representation, questioning sources)
- Connecting mathematical ideas to real world situations and communicating mathematical ideas across a variety of media (e.g., spoken, written, gestured and digital presentations)
- Creating charts, graphs, tables, text, equations, Venn diagrams, trees, decision matrices, etc. as appropriate; using data sets, or other information that is supplied, to create a representation of data, as well as justifying the use of the representation for the audience and context
- Recognizing the role the ordering, area, shape, and space play in a variety of mathematical presentations (e.g., Cartesian graphs, Venn Diagrams, stacked bar graphs, pie charts, tree diagrams, decision matrices, box-and-whisker plots, polar graphs, radar graphs, Gantt charts)
- Identifying patterns, trends and relationships in data, including covariational relationships
- Determining the use and misuse of statistics; understand the difference between correlation and causation

- Building the capacity to read and summarize charts, graphs, tables, and equations and communicate that understanding in multiple ways both spoken and written
- Making and justifying inferences and predictions based on mathematical information represented in a variety of forms
- Using digital tools and platforms (e.g., spreadsheets, videos, chart makers) to support the above goals

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

***Active Learning*:** There is generally not a "one size fits all" procedure that captures the "best" way to present information, nor is their necessarily a single lens through which information might be interpreted. Presenting and interpreting information provides many opportunities for students to devise their own methods of presenting and interpreting, which can serve as an effective means by which to encourage them to select, perform, and evaluate these methods.

***Meaningful Applications*:** Information presentation and consumption is inherently not specific to one context or topic. It is therefore a rich topic through which to emphasize meaningful applications, a key aspect of which is generalization across contexts and topics. Designing experiences for students in which a few key principles of information presentation and consumption emerge as critical across very different contexts could be a productive way to support students in developing these competencies.

***Academic Success Skills*:** Information can be productively presented and interpreted in many different ways, and thus offers an opportunity to promote mathematical creativity and support students' in developing their mathematical confidence in ways not afforded by more "conventional" topics. Identifying and communicating patterns in data (and their associated implications) that are perhaps not initially obvious might help students develop a sense of ownership of the content (effecting a positive change on their own identities as mathematics learners).

Initial resources
Eric Gaze's blog https://thinkingquantitatively.wordpress.com/
Slides from Eric Gaze's presentation for the MIP (see the MIP Box folder or email allison.j.dorko@okstate.edu)

TARGETED TOPIC 2: SPREADSHEETS

Spreadsheets are a useful tool for organizing, analyzing, and visualizing data. Moreover, facility with spreadsheets is useful for students in their real lives and careers, as many occupations involve spreadsheet use. Spreadsheets are particularly useful for Quantitative Reasoning because they allow students to work with large, complex data sets and hence retain more of the complexities of real-world contexts.

Modules developed by a CoRD focusing on spreadsheets **could**:

1. Choose one or two Quantitative Reasoning topics and develop a module for teaching the entire topic with spreadsheets.
2. Choose at least five topics or sub-topics, and develop mini modules for spreadsheet use as part of instruction in those contexts. For example, each mini module might provide a day or two of instructional materials for spreadsheet use to create different types of graphs, do financial math, explore probability, work with statistical analyses, and create mathematical models (e.g., regressions). See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.
3. Propose a new idea that is related to but perhaps not explicitly listed in the topics below.

Generally, modules developed by CoRDs focusing on spreadsheets **should**:

1. Base spreadsheet use in contextual problems, affording students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases)
2. Help students understand the value and utility of spreadsheets for analyzing and solving real-world problems.
3. Give students experience with a variety of spreadsheet uses, such as organizing information, creating formulas and graphs, etc. One such use might be a "what if" analysis, or the process of changing the values in cells to see how doing so affects outcomes. Another such use might be algebraic reasoning through implementing model-based formulas throughout a spreadsheet.
4. Employ spreadsheets to build important ways of thinking in Quantitative Reasoning, such as proportional reasoning, variational reasoning, and critical thinking.
5. Develop the module using google sheets or some other free or easily-accessible technology (e.g., most institutions have an institutional Microsoft Office subscription for students and faculty).
6. As part of the modules the CoRD develops, spreadsheet learning should be accessible via worksheets. While this likely sounds counterintuitive, Gaze (2019) has found this is just as effective – if not more effective – for teaching QR with spreadsheets because it circumvents technical problems and keeps everyone on the same page (which does not always happen when everyone has a computer). Of course, it is also imperative that students work with spreadsheets on a computer, so the modules should strike an appropriate balance of paper and computer.

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

***Active Learning*: **Spreadsheets provide an opportunity for students to explore problematic situations whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned. Because spreadsheets simplify calculations, they allow ease of performing actions, which allows students to focus on justifying their selection of tools and on observing and evaluating the results.

***Meaningful Applications*:** The computational power of spreadsheets allows students to focus on identifying mathematical relationships and making and justifying claims, rather than focusing on performing computations themselves.

***Academic Success Skills*:** Spreadsheets provide an opportunity for students to collaborate and work in groups, which can help build classroom community. They also afford problem exploration and problem-solving. Spreadsheets may be new to students, and because they afford ease of computation, may help lessen students' mathematics anxiety.

Initial resources

Gaze, E. (2019). Thinking quantitatively: Creating and Teaching a Quantitative Reasoning Course. In Tunstall, L., Karaali, G., and Piercey, V. (Eds.), *Shifting Contexts, Stable Core:* Advancing Quantitative Literacy in Higher Education. (p. 89-106). MAA Press.

Pea, R. (1985). Beyond Amplification: Using the Computer to Reorganize Mental Functioning. *Educational Psychologist, 20*, 167-182.

TARGETED TOPIC 3: RATIOS, PROPORTIONS, AND PROPORTIONAL REASONING

Proportional reasoning "involves maintaining a sense of multiplicative scale in a relationship between quantities" (Gaze, 2019, p. 90). It includes understanding proportion, ratio, percent, and linearity; constant multiple/ratio and scaling; constant multiple/ratio of changes and scaling changes; and multiplicative reasoning. Gaze (2019) suggests "the concept of ratio can provide a common theme to convey the interrelated meanings of fractions, percentages, proportions, decimals, and rates" (p. 91). Proportional reasoning helps students make comparisons in real-world contexts, supports covariational reasoning, and assists students in recognizing reasonable answers to numerical problems.

Modules developed by a CoRD focusing on proportional reasoning **could**:

1. Develop a module focused solely on ratio, proportion, percentage, and proportional reasoning (in real-world contexts)
2. Choose at least five Quantitative Reasoning topics or sub-topics and develop proportional reasoning tasks for use as part of instruction in those contexts. See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.
3. Propose a new idea that is related to but perhaps not explicitly listed in the topics below.

Generally, modules developed by a CoRD focusing on proportional reasoning **should**:

1. Base problems about ratio, proportion, and proportional reasoning in contextual problems, affording students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases)
2. Support students in understanding a quantity as a measurable attribute of an object, and a ratio as a relationship between two quantities showing the relative amount of one that is associated with the other
3. Define what it means for a student to understand ratio, percentage, and proportion, and develop a wide variety of tasks to build the concepts of ratio, proportion, percentage, and proportional reasoning *from* real world problems.
4. Support students in recognizing proportional relationships as distinct from nonproportional relationships and justifying why a relationship is/is not proportional
5. Leveraging proportional reasoning to understand covariation as reasoning about two quantities changing simultaneously

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Active Learning: Proportional reasoning tasks afford opportunities for students to explore problematic situations and choose the appropriate tools to describe multiplicative change. Because ratio is a unifying idea, it provides a common structure to connect mathematical ideas.

Meaningful Applications: Proportional reasoning tasks afford opportunities to explore and identify mathematical relationships between quantities. Ratio in its various forms is a common mathematical structure, and CoRDs can create tasks with ratio as a foundation to help students see various forms of ratio as representing the common mathematical structure of multiplicative change.

Academic Success Skills**:** Ratios and proportions can be difficult for students to understand. As such, it affords an opportunity to build students' perseverance. Additionally, because students have possibly struggled with this topic in the past, CoRDs should attend to task design principles that decrease students' mathematics anxiety. Collaborative work can support students in mathematical success in this sometimes difficult topic, and foster classroom community.

Initial resources

Common Core ratio and proportion standards and examples:
http://www.corestandards.org/Math/Content/6/RP/

Gaze, E. (2019). Thinking quantitatively: Creating and Teaching a Quantitative Reasoning Course. In Tunstall, L., Karaali, G., and Piercey, V. (Eds.), *Shifting Contexts, Stable Core:* Advancing Quantitative Literacy in Higher Education. (p. 89-106). MAA Press.

Thompson, P.W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421-456). Reston, VA: NCTM.

TARGETED TOPIC 4: QUANTIFICATION

Understanding a quantity as a measurable attribute of an object is foundational for understanding how to work with numbers and making sense of what numbers represent. It also provides the basis for variational, proportional, and covariational reasoning. Students should be able to quantify a given situation, reason with variables that represent quantities, represent changes in quantities, and make estimates. Students should also understand what it means to measure a quantity, which involves envisioning a measurement process that results in a multiplicative comparison between a targeted magnitude and the unit of measure.

Modules developed by a CoRD focusing on quantification **could**:

1. Develop a module focused solely on quantification
2. Choose at least five Quantitative Reasoning topics or sub-topics (listed below), and develop quantification tasks for use as part of instruction in those contexts. See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.
3. Propose a new idea that is related to but perhaps not explicitly listed in the topics below.

Generally, modules developed by a CoRD focusing on proportional reasoning **should** support students in:

1. Reasoning about quantities in contextual problems, affording students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases)
2. Understanding that a quantity is a measurable attribute of an object
3. Understanding how/why/when to use numerical values rather than qualitative descriptions and why this is important
4. Understanding how numerical values can be visually represented and interpreted (for example, by length, position, area, etc) and how various representations relate to each other
5. Understanding the use of variables for unknown or varying quantities.
6. Developing number sense to estimate, predict, and evaluate the "reasonableness" of numerical answers to context problems.

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

> ***Active Learning*****:** Quantities are constructed in the mind (Thompson, 2011). As such, good quantification tasks necessarily engage students in the mental activity of selecting, performing, and evaluating actions related to quantities and their relationships. Good tasks will expose students to a variety of different quantities and quantitative relationships, and help students see quantitative relationships as common mathematical structures.
>
> ***Meaningful Applications*****:** Quantities are present in all Quantitative Reasoning problems, and as such, helping students understand quantification allows them to identify and interpret meaningful relationships among quantities and to see quantitative relationships as a common and useful mathematical structure.

***Academic Success Skills*:** An understanding of quantities supports student success in many mathematical topics. Building successful experiences for students can help them see themselves as capable doers of mathematics. In the process of those experiences, students can gain perseverance in problem solving, further increasing their mathematics self-efficacy.

Initial resources

Confrey, J., & Smith, E. (1994). Exponential functions, rates of change, and the multiplicative unit. *Educational Studies in Mathematics, 26*(2), 135-164.

Ellis, A. B. (2007). The influence of reasoning with emergent quantities on students' generalizations. *Cognition & Instruction, 25*(4), 439-478.

Lobato, J. & Siebert, D. (2002). Quantitative reasoning in a reconceived view of transfer, *Journal of Mathematical Behavior, 21*, 87–116.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education* WISDOMe Monographs (Vol. 1, pp. 33-57). Laramie, WY: University of Wyoming Press. http://pat-thompson.net/PDFversions/2011QR&Modeling.pdf

TARGETED TOPIC 5: CRITICAL THINKING

Critical thinking involves analysis, evaluation, objectivity and reflection that leads to self-corrective reasoning and the ability to reconstruct an idea from different viewpoints in order to understand a situation, solve a problem, or evaluate an outcome. It enables students to make decisions based on relevant information and make sense of a situation. In addition to the computational and procedural fluency that is involved in arriving at a numerical answer, developing students' critical thinking is essential in a QR course because it involves learning to both ask and answer such questions as "does this make sense?", "what are the implications?", "is this meaningful?", and "what alternative explanations exist?"

Modules developed by a CoRD focusing on quantification **could**:

1. Develop a module focused solely on critical thinking (based in real-world tasks)
2. Choose at least five Quantitative Reasoning topics and design tasks that aim to develop students' capacity for critical thinking with respect to these topics. (See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.)
3. Propose a new idea that is related to but perhaps not explicitly listed in the list below.

Generally, modules developed by a CoRD focusing on quantification **should**:

1. Base problems in contexts that afford students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases). Critical thinking goes hand-in-hand with social contexts in Quantitative Reasoning, as one of the goals is to help students become more knowledgeable citizens.
2. Identify the cognitive habits and characteristics of critical thinkers and how these habits and characteristics might be elicited in instruction.
3. Create tasks that develop students' critical thinking in real-world contexts.

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

***Active Learning*:** Students can rely on critical thinking as they work with problematic situations. In fact, active learning provides a way to operationalize what is meant by "critical thinking": critical thinking is the act in which one devises, selects, performs, and evaluates actions related to a given problematic situation. These "stages" (devise, select, perform, and evaluate) could provide a useful framework for CoRDs as they design tasks that elicit critical thinking.

***Meaningful Applications*:** Critical thinking supports students in transferring mathematical ideas across contexts because it is inherently not specific to one context or topic. Designing experiences for students in which a few key principles of critical thinking (such as intentionally engaging in iterative cycles of devising, selecting, performing, and evaluating) emerge as essential across very different contexts could be a productive way to support students.

***Academic Success Skills*:** Critical thinking is a key habit of mind that supports students in mathematical success, and as such, can build students' growth mindsets and make them feel capable of doing and learning mathematics. Because critical thinking often requires critiquing others' reasoning, it is an ideal tool in building a classroom community. (Of course, critiquing others' reasoning must be done respectfully; it is therefore suggested that CoRDs proposing such activities carefully consider how they might be framed and implemented in a way that contributes, and is not detrimental to, the classroom community.)

Initial resources

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational studies in Mathematics*, *58*(1), 45-75.

TARGETED TOPIC 6: MODELING

Modeling is a powerful tool for analyzing real-life phenomena. Modeling is the process of using mathematics to describe, analyze, and gain insight into real life phenomena. It entails identifying and representing quantities and determining relationships among relevant quantities. Modeling requires careful recognition of, and attention to, the relevant quantities involved in the situation.

Modules developed by a CoRD focusing on modeling **could**:

1. Develop a module focused solely on modeling (based in real-world contexts)
2. Choose at least five Quantitative Reasoning topics or sub-topics, and develop modeling tasks for use as part of instruction in those contexts. See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.
3. Propose a new idea that is related to but perhaps not explicitly listed in the list below.

Generally, modules developed by a CoRD focusing on modeling **should:**

1. Base modeling problems in contextual problems, affording students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases)
2. Design tasks that encourage students to develop clear (mental and physical) images of a problem scenario to identify relevant quantities and relationships among them
3. Support students in understanding the meaning of model parameters, how those parameters impact important outcomes, and how the parameters are developed (e.g., a qualitative understanding of how one arrives at the slope in a regression calculation)
4. Give students experience in modeling with technology (e.g., spreadsheets, interactive mathematical software, graphing calculators), and leverage technology to give students experience with modeling messy, complicated, real-life data
5. Give students experience with both constructing models and analyzing given models

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

***Active Learning*:** Modeling provides an opportunity for students to reason with complex data. As such, it engages students in problematic situations, as well as selecting, performing, and evaluating the type of model to use.

***Meaningful Applications*:** Models support students in identifying mathematical relationships and making and justifying claims. In particular, using technology for calculations affords a focus on relationships, exploration, and making and justifying claims.

***Academic Success Skills*:** A key component of modeling is imposing a structure on a situation in order to identify patterns and draw implications from a situation. Modeling can therefore elicit students' creativity in ways that are not commonly afforded by other mathematics topics or courses (thereby increasing confidence and potentially positively influencing their identities as mathematics learners). The use of technology, which reduces the focus on students' computations in favor of other skills, might help lessen students' mathematics anxiety.

Initial resources

Gaze, E. (2019). Thinking quantitatively: Creating and Teaching a Quantitative Reasoning Course. In Tunstall, L., Karaali, G., and Piercey, V. (Eds.), *Shifting Contexts, Stable Core:* Advancing Quantitative Literacy in Higher Education. (p. 89-106). MAA Press.

Thompson, P. W. (2011). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education.* WISDOMe Mongraphs (Vol. 1, pp. 33- 57). Laramie, WY: University of Wyoming. Available at http://bit.ly/2kJv9fy.

TARGETED TOPIC 7: PROBLEM SOLVING

A problem is generally considered to be a mathematical task for which one does not know a solution method in advance. Problem solving therefore generally involves the mental activity in which one must engage to identify, select, perform, and evaluate solution methods. Developing productive problem-solving habits and a disposition for critical thinking are essential components of mathematical proficiency. Problem-solving skills are necessary to aid the critical thinking on which quantitative reasoning relies. A goal of entry-level mathematics instruction is to enhance students' problem-solving ability while leveraging it as a foundation for their learning of central ideas.

Modules developed by a CoRD focusing on problem solving **could**:

1. Develop a module focused solely on problem solving (based in real-world tasks)
2. Choose at least five Quantitative Reasoning topics and design tasks that aim to develop students' capacity for problem solving with respect to these topics. (See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.)
3. Propose a new idea that is related to but perhaps not explicitly listed in the topics below.

Generally, a CoRD focused on problem exploration and solving **should**:

- Propose contextual problems, affording students an opportunity to address the social context of the data (e.g. including multiple values/perspectives/needs, attending to implicit biases). Because Quantitative Reasoning problems are posed in real world context, the context plays a significant role in decision making.
- Identify the cognitive habits and characteristics of effective problem solvers.
- Attend specifically to ways to help students explore and make sense of problems (e.g., focus on understanding the question, organizing and making sense of the information given, processing, asking what other information is needed and what information, if any, is unnecessary, thinking about what form an answer to the given question could take (e.g. a number, set of numbers, equation, graph, explanation, etc).
- Articulate instructional design principles for good problem solving tasks, and use them to create tasks in one of the items below:
    - Develop a module focused solely on problem solving (based in real-world contexts, focusing on one topic within Quatitative Reasonoing)
    - Choose at least five topics or sub-topics, and develop modeling tasks for use as part of instruction in those contexts. See Appendix A for a list of topics frequently covered in Oklahoma Quantitative Reasoning courses.
    - Propose something different than the above, which are intended to be starting points for ideas.

Participants of the MIP Workshop on QR suggested the following ways modules for this targeted topic could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Active Learning**:** Because problem solving involves identifying and outlining methods of approaching mathematical situations (for which no method is known ahead of time), problem solving tasks inherently afford opportunities for students to engage in cyclic, iterative processes of identifying, selecting, performing, and evaluating actions. CoRD members could potentially explore the relationship of this process to problem solving frameworks that are outlined in the literature.

Meaningful Applications**:** Problem solving tasks afford an opportunity for students to identify mathematical relationships, make and justify claims, and generalize across contexts because the nature of problem solving are inherently not context-specific. CoRD members are therefore encouraged to identify context-invariant aspects of problem solving that students themselves might be supported in attending to and developing an awareness of as they engage in various tasks.

Academic Success Skills**:** Problem solving – that is, when one is devising a solution to a situation in which the method for obtaining a is not known in advance – is related to a number of key academic success skills, including perseverance (often the first few methods attempted are not successful), mathematical identity and confidence (seeing a difficult, multifaceted problem through to its solution can support positive self-images related to mathematics), and classroom community building (collaborating and sharing ideas to help devise a solution method). CoRD members are encouraged to provide some specific guidance to how such academic success skills might be fostered.

Initial resources

Carlson, M. P., & Bloom, I. (2005) The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics, 58*(1), 45-75.

Silver, E. A. (Ed.). (2013). *Teaching and learning mathematical problem solving: Multiple research perspectives.* Routledge.

Thompson, P. W. (1985). Experience, problem solving, and learning mathematics: Considerations in developing mathematics curricula. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 189-243).

Wilkersen-Jerde, M. & Wilensky, U. (2011). How do mathematicians learn math?: Resources and acts for constructing and understanding mathematics. *Educational Studies in Mathematics, 78*(1), 21-43.

APPENDIX A

Alphabetical List of Specific Topics in Quantitative Reasoning Courses in Oklahoma

This list was generated from a question on the Initiation Workshop application asking participants what textbooks their QR course uses and what topics they cover. As such, it is a collective list of QR topics taught at Oklahoma universities.

Algebra basics

Art and math ,tivwtigxmzi0$w}qqixv}0$tvstsvxmsr0$kspHir$vexms0$xiwwippexmsrw0$qywmg-$

Counting

Cryptography

Financial math

Fractals

Functions (linear, exponential, logarithmic, quadratic)

Geometry

Graph theory

Historical counting systems

Logic

Modular arithmetic

Number theory

Probability

Problem solving

Rates, ratio, proportion, percentages

Set theory

Sports and math

Spreadsheets

Statistics

Voting theory, weighted voting, apportionment