The Mathematical Inquiry Project

Request for Proposal

Collaborative Research and Development

College Algebra and Precalculus

The Mathematical Inquiry Project (MIP) is a statewide collaboration among mathematics faculty in Oklahoma to improve entry-level undergraduate mathematics instruction through three guiding principles:

Active Learning: Students engage in active learning when they work to resolve a problematic situation whose resolution requires them to select, perform, and evaluate actions whose structures are equivalent to the structures of the concepts to be learned.

For more information on the MIP Active Learning Principle, visit https://okmip.com/active-learning/

Meaningful Applications: Applications are meaningfully incorporated in a mathematics class to the extent that they support students in identifying mathematical relationships, making and justifying claims, and generalizing across contexts to extract common mathematical structure.

For more information on the MIP Meaningful Applications Principle, visit https://okmip.com/applications/

Academic Success Skills: Academic success skills foster students' construction of their identity as learners in ways that enable productive engagement in their education and the associated academic community.

For more information on the MIP Academic Success Skills Principle, visit https://okmip.com/academic-success-skills/

Description of CoRD modules

CoRD modules should be designed to promote all three of the MIP components of inquiry: active learning, meaningful applications, and academic success skills. An overview of the module should articulate explicitly how these three components are supported.

In order to communicate the CoRD's approach to developing the targeted concepts to faculty using the MIP resources, modules should include an analysis of its primary conceptual goals. This analysis should include details such as the ways of understanding desired as an outcome for all students in the course, common entry points for students' understanding (including relevant supporting concepts), a progression of challenges and solutions that students should engage through the module to develop these understandings, common pitfalls in the learning process and ways to address them, and a description of ways in which these ideas support thinking and learning throughout the entire course.

The core of a module will be a set of instructional materials. The MIP seeks to support development of modules for entry-level college mathematics courses that develop targeted concepts as a unifying topic throughout the course. Consequently, the materials in a module will not typically consist of a sequential series of lessons, but rather provide broader instructional resources to be used throughout the course.

These materials should include assessment materials that allow an instructor both to assess how their students have progressed relative to the targeted goals and to identify ways to improve their own instruction.

As corequisite remediation for entry-level college mathematics is a critical reform in the state of Oklahoma, modules should include a description of how it would be implemented differently in a corequisite class, including any additional resources necessary to do so.

After a successful review the CoRD will pilot the module with a class or group of students and incorporate a description of test implementation and its results, a discussion of the refinements and recommendations made based on test implementation, and short video clips with commentary to illustrate effective implementation.

Review and Revision

Once a CoRD submits a module, it will be reviewed by at least two other faculty with expertise in the topic to inform an editorial decision of "Accept," "Accept with minor revision," "Revise and resubmit," or "Reject," along with directions for revision if appropriate. After a favorable review, the CoRD will revise and pilot their module, incorporating feedback gained during the review process and submit a final module for publication on the project website.

Author Stipends

Each author in the CoRD will receive a $2500 stipend after delivery of a complete initial draft of the module and an additional $1000 stipend after delivery of a complete revision of the module based on the editorial decision.

Opportunities for leading regional workshops and mentoring

The MIP will leverage faculty leadership and expertise developed through its Initiation Workshops and CoRDs to also develop and deliver 40 institutional and regional professional development workshops, across the state of Oklahoma, on teaching the new courses, incorporating applications and active learning with the modules, and addressing academic success skills. Each Regional Workshop will last a full day and engage approximately 20 mathematics faculty in implementing one or two of the modules developed by the CoRDs and ensuring familiarity with the module resources. Each workshop will be led by faculty from the respective CoRDs with support of MIP personnel who will also assist the leaders in designing the workshop activities with advice from project consultants. A goal of the Regional Workshops will be to engage all relevant faculty in hosting at nearby institutions and to develop a structure that will provide training for new faculty and continuing professional development for all faculty.

The MIP will also support 425 semester-long faculty mentoring relationships between CoRD leaders and one or two faculty who are first implementing MIP resources in a class they are teaching. A goal of these mentoring relationships is to develop institutional and regional communities whose members meet regularly and reinforce and support the cultural practices necessary for mathematics learning through inquiry.

Proposal requirements

The MIP seeks to support the development of modules on the following targeted topics. See the following pages for details of each of these topics.

- Rate of change and covariation
- Functions and their fundamental characteristics
- Multiple problem-solving strategies and representational equivalence
- Quantitative reasoning and modeling

Proposals should include each of the following:

1. A cover page designating which of the targeted topics the proposed CoRD will address, the entry-level college course(s) for which it will develop instructional resources, names of all proposed CoRD members (3-5 people), their institutions, email addresses, and phone numbers.

2. The CoRD's initial image of how to develop the targeted concept as a unifying topic throughout the entry-level course.

3. The CoRD's initial plan to promote all three of the MIP components of inquiry: active learning, meaningful applications, and academic success skills, in their module.

4. A description of prior experience of each CoRD member relevant to their development of the proposed module.

Proposal Length

The full text of a proposal should not exceed 2,000 words.

Consultation

The MIP encourages discussions with any of the project team on the planning and preparation of a proposal. Throughout the CoRD's work, MIP project personnel will provide associated resources and advice. The MIP will also organize events throughout the year to allow multiple CoRDs to present their progress and discuss ways to benefit from and integrate their approaches.

Proposal Submission

Completed proposals should be emailed to William (Bus) Jaco at william.jaco@okstate.edu. We strongly encourage discussions with the project team to avoid proposing work on topics that have already been assigned a CoRD. The MIP generally responds to proposals within one month of their submission. During the review of proposals, the MIP may request additional information or modifications before approval.

Proposals for College Algebra and Precalculus CoRDs may be submitted on a continuing basis until July 31, 2020 with work typically extending up to six months from the start date. Variations on topics and timing may be arranged through individual discussions with the MIP project leadership.

College Algebra and Precalculus Targeted Topics

Rate of change and covariation

A critical foundation for reasoning about rates of change is conceiving of changes in quantities as quantities in their own right and distinguishing such changes from the original quantities. From this foundation, students may begin to understand, distinguish, and use the meanings of constant rate of change and average rate of change in various contexts and representations. Students in College Algebra and Precalculus should develop a robust ability to articulate, distinguish, and use the meanings of constant and average rates of change. In particular, constant rate of change entails a proportional relationship between changes in the two quantities (e.g., see the conceptual analysis in Thompson, 1994). Reasoning about these changes and their proportional relationship across multiple representations can build an important foundation for further development of average and instantaneous rates.

Participants of the MIP Workshop on College Algebra and Precalculus suggested development of modules addressing the following areas:

1. Help students conceive of changes in quantities as meaningful quantities in their own right (e.g., see early tasks involving describing and reasoning about changes in quantities in Carlson, Oehrtman, & Moore, 2016).
2. Engage students in interpreting average rates of change as a constant rate for an auxiliary scenario with the same total changes in both quantities. These materials could reinforce and motivate the use of function notation in algebraic representations of average rates, developing the difference quotient.
3. Informally introduce instantaneous rates through a context that necessitates finding average rates over progressively smaller intervals.
4. Unpack rate of change statements in terms of coordinating amounts of change. Such tasks may ask students to analyze amounts of change in the function for constant increments of the independent variable (e.g., see MA3 reasoning in Carlson et al., 2002).
5. Draw diagrams that represent changes in the output variable corresponding to successive increments in the input variable to help students conceptualize varying rates more robustly. Students should subsequently represent these relationships graphically and algebraically and interpret them in terms of rate of change in the problem context.

Participants of the MIP Workshop on College Algebra and Precalculus suggested the following ways modules could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Meaningful Applications: Students should engage in rates of change as a natural entry point to understand, represent, and explain, how quantities change in actual situations. Correspondingly, identifying and applying key rate of change characteristics of various function types can help reinforce broader understanding of these functions and their value in appropriate modeling scenarios. Varying the contexts promotes students' development of a generalized concept of rate of change that is not bound to any single situation or representation.

Active Learning: Students in a College Algebra or Precalculus course will have significant experience applying rate of change tools in proceduralized ways. Thus, it is particularly important that modules engage students in tasks that challenge these rote applications and require them to explore the underlying meanings, especially in terms of relationships between amounts of change.

Academic Success Skills: Exploring rate of change in-depth and in meaningful applications can help students reinforce their identity as STEM students. Modules could also attend to reinforcing a growth mindset and persistence by providing scaffolding that keeps students engaged without preempting their ability to develop significant solutions on their own.

References

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education, 33*(5), 352–378.

Thompson, P. W. (1994). The development of the concept of speed and its relationship to concepts of rate. In G. Harel & J. Confrey (Eds.), T*he development of multiplicative reasoning in the learning of mathematics* (pp. 179–234). Albany, NY: SUNY Press.

Functions and their fundamental characteristics

Many student difficulties in reasoning with functions are based in a static conception tied to evaluating a function one step at a time, typically tied to the formula. This is often called an "action view," and renders reasoning dynamically or about multiple values at a time nearly impossible. In contrast, a "process view" of function in which a student can conceive of the entire process happening to all input values at once, and is thus able to conceptually run through a continuum of input values while attending to the resulting impact on output (e.g., see the discussion of action and process views in Oehrtman, Carlson, & Thompson, 2008).

Participants of the MIP Workshop on College Algebra and Precalculus suggested development of modules addressing the following areas:

1. Ask students to coordinate multiple function processes (e.g., through composition, addition, or in defining an increasing function).
2. Ask students about the behavior of functions on entire intervals in addition to single points (e.g., describing a function's behavior as input values increase continuously through the domain or finding the image of an interval)
3. Ask students to reverse a function processes (e.g., finding the preimage of a specified output value or interval).
4. Ask students to make and compare judgments about functions across multiple representations.
5. Help students understand, explore, apply, and contrast the fundamental features of various classes of functions including linear, exponential, quadratic, polynomial, logarithmic, rational, trigonometric, and radical functions. Important characteristics to emphasize include, but are not limited to, domain, range, zeroes, extrema, limiting behavior, increasing, decreasing, periodic behavior.

Participants of the MIP Workshop on College Algebra and Precalculus suggested the following ways modules could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Meaningful Applications: Modules could emphasize modeling and interpretation to reinforce functions as a tool to describe the world. The coordination of two quantities and univalence built into the mathematical structure of functions can gain compelling meaning from natural relationships and constraints between quantities in real world situations. One may ask students to contrast the domain and range of functions based on the problem context with the domain and range derived from algebraic constraints alone. The concept of functions and function notation can be motivated and reinforced by engaging students in reasoning with and expressing quantities determined through correspondence, such as, $\Delta h = h(V+\Delta V) - h(V)$. Students should also identify and interpret key parameters in each function class in terms of the context in which it is being applied and in its various mathematical representations.

Active Learning: Students should be engaged in tasks that go beyond treating functions as equations and provides opportunities for them to create functions appropriate to solve novel problems and invoke function notation in ways responsive to that problem-solving activity

Academic Success Skills: When improperly motivated, introduction of functions can seem arbitrary and unnecessarily complicated, raising a barrier to many students. Modules should help students become confident in their use of functions as a foundation of the language of mathematics and science.

References

Oehrtman, M., Carlson, M., & Thompson, P. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. In M. Carlson & C. Rasmussen (Eds.), *Making the Connection: Research and Practice in Undergraduate Mathematics, MAA Notes, Volume 73*, 27-41. Washington, DC: Mathematical Association of America.

Multiple problem-solving strategies and representational equivalence

Students will be required to apply the skills learned in College Algebra and Precalculus in an extremely diverse range of ways, meaning that they must also develop flexible problem-solving strategies related to these skills. MIP modules can foster an image of algebraic skills as general tools and an ability to strategically select, apply, and reflect on their application in a wide variety of goal-oriented activity. Additionally, access to multiple approaches enables a broader range of meaningful student participation and enables students to select approaches that work better for them, to apply multiple reinforcing approaches, or to check reasoning through alternate methods.

Participants of the MIP Workshop on College Algebra and Precalculus suggested development of modules addressing the following areas:

1. Develop general problem-solving strategies, such as drawing a diagram of a situation and representing all relevant quantities in the diagram, drawing a graph of a quantitative relationship and representing all relevant quantities in the graph, reviewing underlying concepts and terminology, attempting a numerical or approximate approach, looking for counterexamples, communicating one's problem-solving process to another person, and looking for equivalent forms or representations.

2. Develop content-specific problem-solving strategies, such as factoring or expanding, applying a Pythagorean identity, multiplying by a clever form of 1 or adding a clever form of 0, rationalizing an expression, and reducing trigonometric expressions to sines and cosines.
3. Develop the ability to apply algebraic skills in novel situations. Modules will need to help students recognize structural equivalence between problem settings and known tools.
4. Develop an understanding of why algebraic procedures work, thus promoting their meaningful interpretation and recall.
5. Develop an image of equivalence of algebraic expressions under various algebraic operations.
6. Develop an image of the invariance of solution sets of equations or a system of equations under various algebraic operations.

Participants of the MIP Workshop on College Algebra and Precalculus suggested the following ways modules could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Meaningful Applications: Applications are beneficial for developing problem-solving strategies and reasoning with representational equivalence, as they provide opportunities to express mathematical problems in multiple, non-routine contexts. As students begin to see the similarity of structure of situations and their algebraic tools across multiple settings, they are able to begin developing a generalization of the mathematics independent of the settings in which they were originally experienced.

Active Learning: It is essential to provide students with tasks that are genuine problems, beyond their current repertoire of familiar procedures, to provide them the genuine opportunity to activate problem-solving strategies. Modules should focus on helping students deliberately attend to all phases of problem-solving, including planning and evaluation as well as carrying out solution methods. Having them explicitly reflect on their partial or completed solution processes can also help reify productive strategies for future use.

Academic Success Skills: Students should become increasingly open to productive struggle. Many students, even those who have been highly successful in previous mathematics classes, view any difficulty as an indication of their lack of ability. Viewing challenging tasks as a natural part of mathematical tasks, and even as a necessary aspect of learning, can help students develop persistence (e.g., see the framework involving steps in the cyclic problem-solving process and the impact of underlying beliefs and affect in Carlson & Bloom, 2005)

References

Carlson, M., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem solving framework. *Educational Studies in Mathematics*, *58*, 45–75.

Quantitative reasoning and modeling

The MIP aims to support modeling in College Algebra and Precalculus primarily through student activity to mathematically represent quantities and quantitative relationships and to manipulate or interpret these representations to draw inferences about the context. Modeling involves the ability to “decontextualize (to abstract a given situation, represent it symbolically, and manipulate the resultant symbols as if they have a life of their own) and to contextualize (to pause as needed during the manipulation process in order to probe into the referents for the symbols at hand)" (Common Core Practice Standards in the Common Core State Standards Initiative, 2010). Contextualizing also involves "interpreting the results in the context of a situation and reflecting on whether the results make sense, possibly improving the model if it has not served its purpose" (Common Core Practice Standards in the Common Core State Standards Initiative, 2010).

Participants of the MIP Workshop on College Algebra and Precalculus suggested development of modules addressing the following areas:

1. Help students conceive and describe real-world quantities through appropriate mathematical representations. Contexts should be chosen to make the mathematics amenable to students' intuitive reasoning that can subsequently be represented more mathematically by variables, expressions, diagrams, and graphs.

2. Help students conceive and describe relationships between quantities through appropriate mathematical representations. Again, contexts should be chosen to enable students to more intuitively state, justify, or question relationships between quantities, before expressing them through mathematical representations.

3. Help students generalize context-specific reasoning by exploring the same underlying mathematical structure in multiple contexts, then reflecting on the similarities and differences across the resulting models (e.g., see the description of a learning trajectory across calculus leveraging abstraction across multiple contexts in Oehrtman, 2008).

4. Help students abstract mathematical structure by applying concepts developed earlier tasks as tools for making sense of new situations in later tasks (e.g., see the description of levels of emergent models in Gravemeijer, Cobb, Bowers, & Whitenack, 2000).

5. Develop working with quantities as a central habit of mind for students. This includes approaching any modeling situation with the initial aim to identify the relevant quantities for the given goal (e.g., see the discussion of extensive quantification in Thompson, 1994). Students should then distinguish between constant and variable quantities and identify relationships between these quantities determined by the situation. Many students will need help articulating these relationships initially using concrete numerical values for specific variable quantities, then seeing the algebra as a generalization of the multiple arithmetic expressions generated by choosing different values.

6. Help students draw effective diagrams of situations with the appropriate information and level of detail to support mathematical modeling.

7. Help students model changes in quantities and rates of change of one quantity with respect to another. This modeling should i) reinforce a concept of changes in quantities as meaningful quantities in their own right, ii) develop a quantitative conception of rate of change, and iii) help students identify rate of change features in contexts that correspond to particular function types to choose an appropriate algebraic form of a model (e.g., see examples of tasks involving modeling with changes and rates of change in Carlson, Oehrtman, & Moore, 2016).

8. Emphasize linear, exponential, and quadratic models that reinforce key quantitative concepts of constant rate of change, rate proportional to amount, and constant acceleration, respectively.

Participants of the MIP Workshop on College Algebra and Precalculus suggested the following ways modules could address the three MIP components of mathematical inquiry (see descriptions of these components at https://okmip.com):

Meaningful Applications: Although modeling essentially involves coordinating meanings between real-world contexts and mathematical objects and relationships, not all modeling activity productively develops conceptual understanding. In particular modules should focus students on identifying common structure across multiple modeling activities with different contexts as the source of abstracting the particular mathematical concept(s) common to them all.

Active Learning: Modules should engage students in developing, applying, and interpreting models at all stages. In doing so, they must transfer meaning both from context to mathematical representations and vice-versa.

Academic Success Skills: Modules should help students develop a view that mathematics is meaningful, both as a set of tools to model real-world situations, but also in the abstract, as generalizations of structures present across a wide variety of contexts. Students' engagement in this process should develop their own agency in creating these meanings and reinforce their ability to learn through persistence.

References

Council of Chief State School Officers & National Governors Association Center for Best Practices (2010). *Common core state standards for mathematics.* Common Core State Standards Initiative. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf.

Carlson, M., Oehrtman, M., & Moore, K. (2016). *Precalculus, Pathways to Calculus: A Problem Solving Approach*, Sixth Edition. Phoenix, AZ: Rational Reasoning.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, Modeling, and Instructional Design. In Paul Cobb, Erna Yackel, & Kay McClain (Eds.) *Symbolizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design*. Mahwah, NJ: Erlbaum and Associates. 225-273.

Oehrtman, M. (2008). Layers of abstraction: Theory and design for the instruction of limit concepts. In M. P. Carlson & C. Rasmussen (Eds.), *Making the Connection: Research and Teaching in Undergraduate Mathematics Education,* (MAA Notes, Vol. 73, pp. 65-80). Washington, DC: Mathematical Association of America.

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